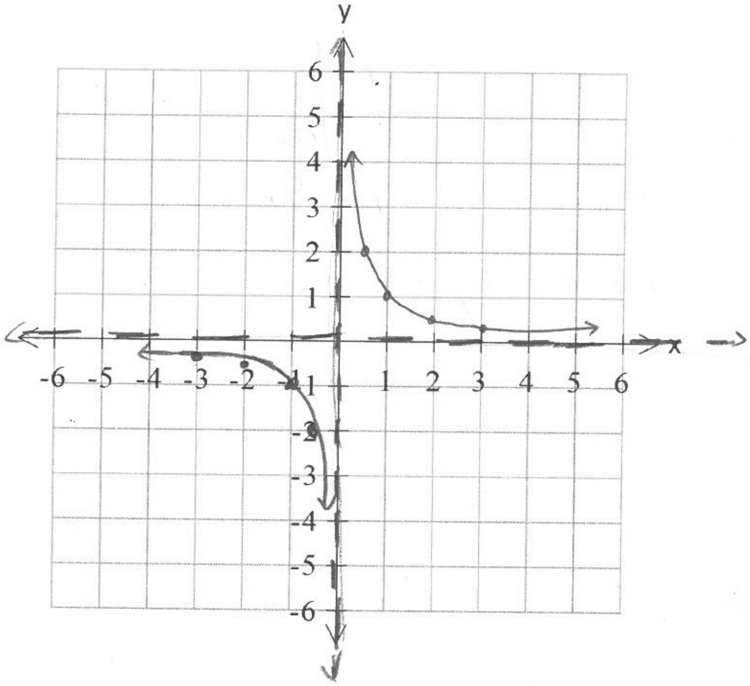
Relation between 2 quantities,

Lesson 4: Identifying Characteristics of Rational Functions

A rational function has the form $y = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both polynomial expressions $q(x) \neq 0$.

Non-permissible values are values of x that would make the denominator equal zero.

Example 1: Graph $y = \frac{1}{x}$ using a table of values.



x	y
-3	$-\frac{1}{3} = -0.\overline{33}$
-2	$-\frac{1}{2} = -0.5$
-1	-1
$-\frac{1}{2}$	-2
0	undefined.
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2} = 0.5$
3	$\frac{1}{3} = 0.\overline{3}$

Characteristic	$y = \frac{1}{x}$
Non-permissible value	$x \neq 0$
Equation of vertical asymptote	$x = 0$
Equation of horizontal asymptote	$y = 0$
Domain	$(-\infty, 0) \cup (0, \infty)$ OR $x \neq 0$
Range	$(-\infty, 0) \cup (0, \infty)$ OR $y \neq 0$

Notes on asymptotes:

1. Vertical Asymptote and Hole

Scenario A) the numerator & denominator have no common factor; so there are vertical asymptote(s)

Scenario B) The numerator & denominator have common factor, so the function can be simplified. There will be a hole (Point of discontinuity).

2. Horizontal and Oblique asymptote.

* Numerator & denominator do not have common factors.

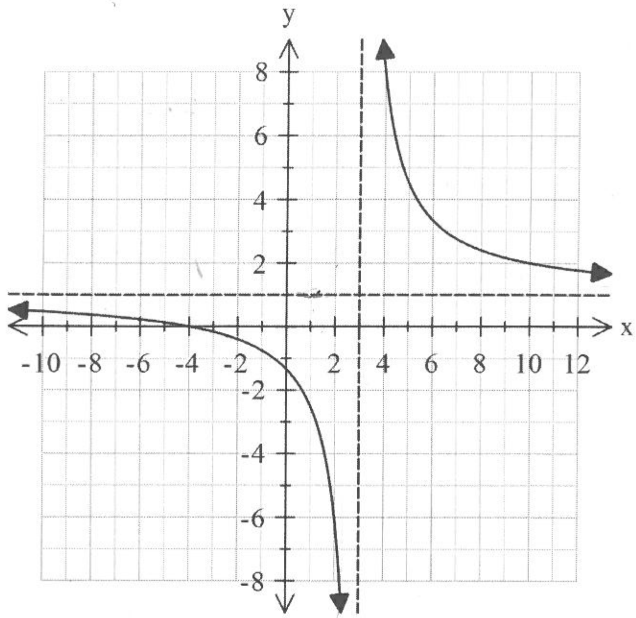
Scenario A) $\deg f(x) < \deg g(x)$ then $y=0$ is the horizontal asymptote

Scenario B) $\deg f(x) = \deg g(x)$ then horizontal asymptote is at $y=\frac{a}{b}$ where "a" lead coeff. of $p(x)$ and "b" lead coeff of $g(x)$.

Scenario C) degree $f(x)$ is $>$ $g(x)$ by 1. Then there is an oblique asymptote. We ~~[illegible]~~ have to perform a synthetic division.

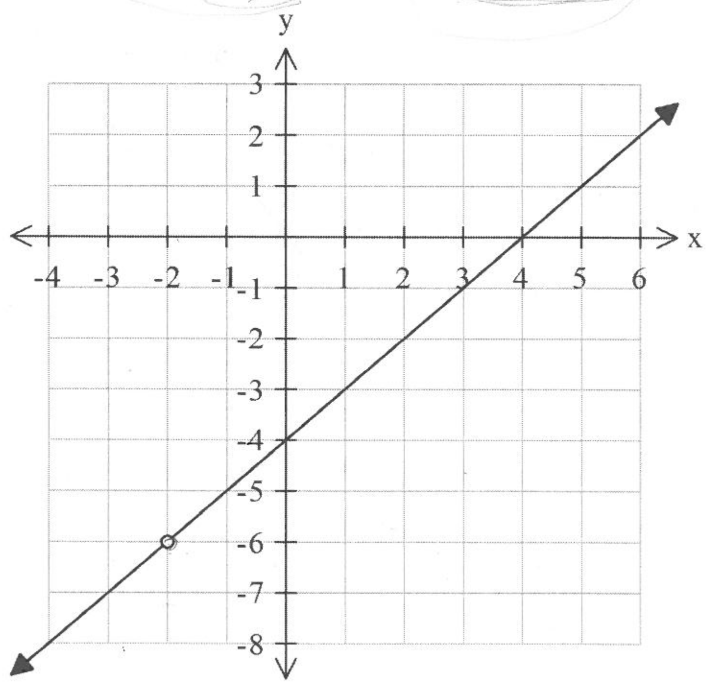
Example 2: State the characteristics of the following rational functions.

a) $y = \frac{x+4}{x-3}$ ← vertical asympt



Characteristic	$y = \frac{x+4}{x-3}$
Non-permissible value	$x - 3 \neq 0$ $x \neq 3$
Equation of vertical asymptote	$x = 3$
Equation of horizontal asymptote $y = \frac{a}{b} = \frac{1}{1}$	$\deg f(x) = \deg g(x)$ $y = 1$
Domain	$(-\infty, 3) \cup (3, \infty)$
Range	$(-\infty, 1) \cup (1, \infty)$
Coordinates of hole(s)	No hole

b) $y = \frac{(x-4)\cancel{(x+2)}}{\cancel{x+2}}$ $y = x - 4$



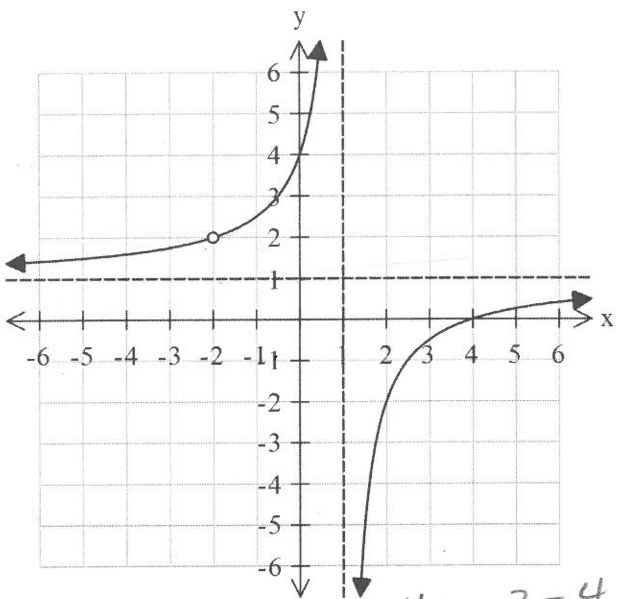
Characteristic	$y = \frac{(x-4)(x+2)}{x+2}$
Non-permissible value	$x + 2 \neq 0$ $x \neq -2$
Equation of vertical asymptote	NO vertical asymp because we cancelled the common factor $x+2$
Equation of horizontal asymptote	No horiz. asympt
Domain	$(-\infty, -2) \cup (-2, \infty)$ or $x \neq -2$
Range	$(-\infty, -6) \cup (-6, \infty)$ or $y \neq -6$
Coordinates of hole(s)	$(-2, -6)$

$y = x - 4$

$y = -2 - 4$

$y = -6$

c) $y = \frac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$ $\quad y = \frac{1x-4}{1x-1}$

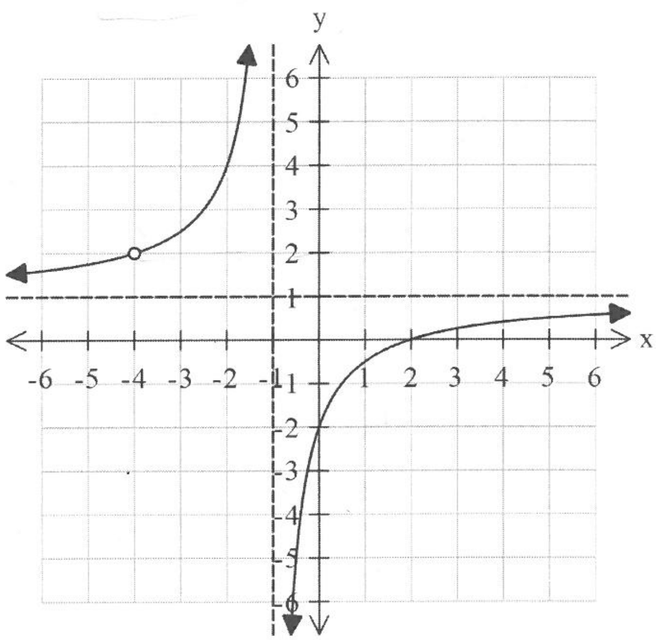


Characteristic	$y = \frac{(x-4)(x+2)}{(x+2)(x-1)}$
Non-permissible values	$x \neq -2$ Hole $\quad x = +1$ vert. asymp
Equation of vertical asymptote	$x = 1$
Equation of horizontal asymptote	$y = \frac{a}{b} = \frac{1}{1} = 1$ $\quad y = 1$
Domain	$x \neq -2,\ x \neq 1$
Range	$y \neq 1,\ y \neq 2$
Coordinates of hole(s)	$(-2, 2)$

$y = \frac{-2-4}{-2-1}$

$y = \frac{-6}{-3} = 2$

d) $y = \frac{x^2+2x-8}{x^2+5x+4}$



Characteristic	$y = \frac{x^2+2x-8}{x^2+5x+4}$
Non-permissible values	$x+4 \neq 0$ $\quad x = -4$ (HOLE) $\quad x = -1$ vert. asymp
Equation of vertical asymptote	$\boxed{x = -1}$
Equation of horizontal asymptote deg f(x) = deg g(x)	$y = \frac{a}{b} = \frac{1}{1}$. $\boxed{y = 1}$
Domain	$x \neq -4$ and -1
Range	$y \neq 1$ and 2
Coordinates of hole(s)	$(-4, 2)$

$y = \frac{x^2+2x-8}{x^2+5x+4}$

$y = \frac{\cancel{(x+4)}(x-2)}{\cancel{(x+4)}(x+1)}$

Example 3: Use the equations of the following rational functions to determine non-permissible values, equations of any vertical and horizontal asymptotes, and coordinates of any hole.

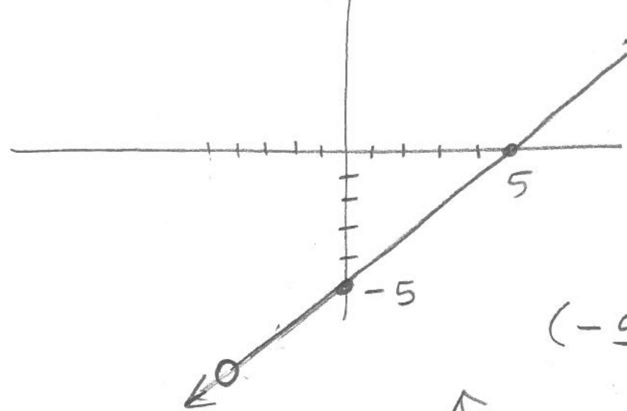
a) $y = \frac{x^2-25}{x+5}$ $y = x - 5$

No Horizontal & Vertical asymptotes

The coord. of hole $x = -5$

$y = \frac{(\cancel{x+5})(x-5)}{(\cancel{x+5})}$

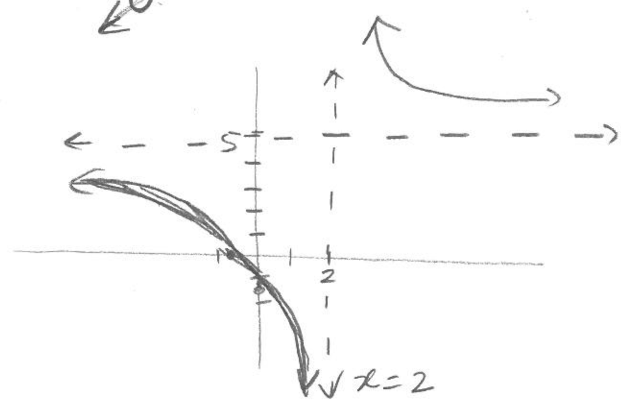
n.p.v $x+5 \neq 0$
$x \neq -5$ ✓
(HOLE)



$y = x - 5$
$y = -5 - 5$
$y = -10$
$(-5, -10)$

b) $y = \frac{5x+3}{x-2}$

npv: $x \neq 2$
vertical asymptote at $x = 2$
Horizontal asymptote at $y = \frac{5}{1} = 5$



c) $y = \frac{x^2-x-2}{x+1}$

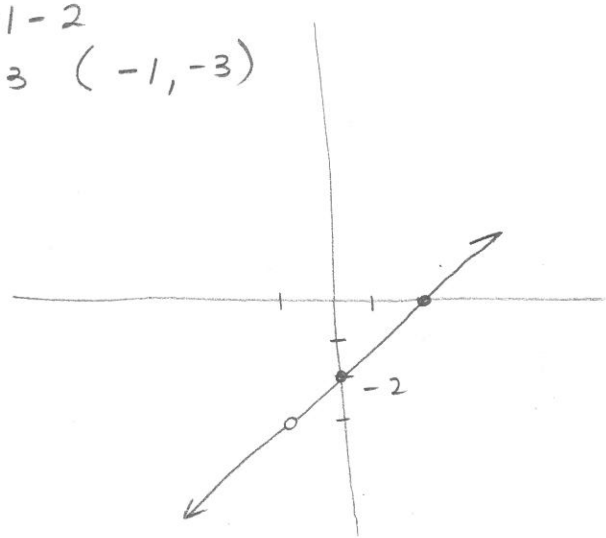
$y = \frac{(x-2)(\cancel{x+1})}{(\cancel{x+1})}$

The hole is at
$y = x - 2$
$y = -1 - 2$
$y = -3$ $(-1, -3)$

n.p.v ⇒ $x+1 \neq 0$
$x \neq -1$

Since $(x+1)$ cancels out $x \neq -1$ is a HOLE

the simplified function is $y = x - 2$ which is an equation of a line



No horizontal asymptote.