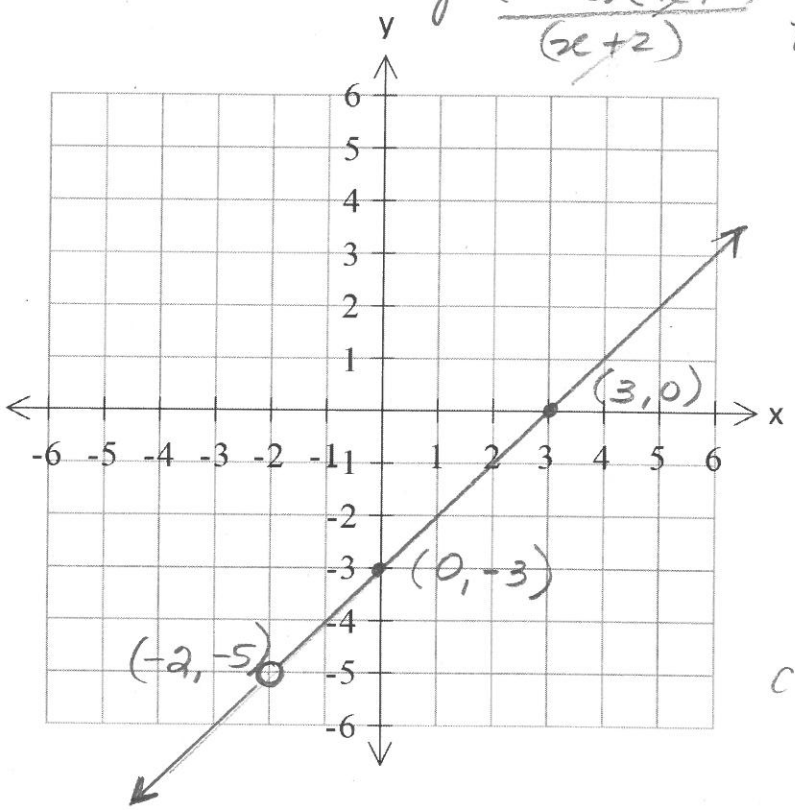
Lesson 5: Sketching the Graph of a Rational Function

To sketch the graph of a rational function, determine and state:

- The non-permissible values of x (holes and vertical asymptotes).
- The equation of the horizontal asymptote (if there is one.)
- The intercepts.

Example 1: Sketch the graph of $y = \frac{x^2-x-6}{x+2}$

$y = \frac{(x-3)(x+2)}{(x+2)}$



npv: $x+2 \neq 0$

$x \neq -2$

There is a hole at $x=-2$

The coordinate of the hole:

$y = x - 3$

$y = -2 - 3$

$y = -5$

HOLE is at $(-2, -5)$

No vertical asymptote.

deg $p(x)$ > deg $q(x)$

$y = x - 3$

No Horizontal asymptote.

calculate y-int, set $x=0$

$y = 0 - 3$

$y = -3$

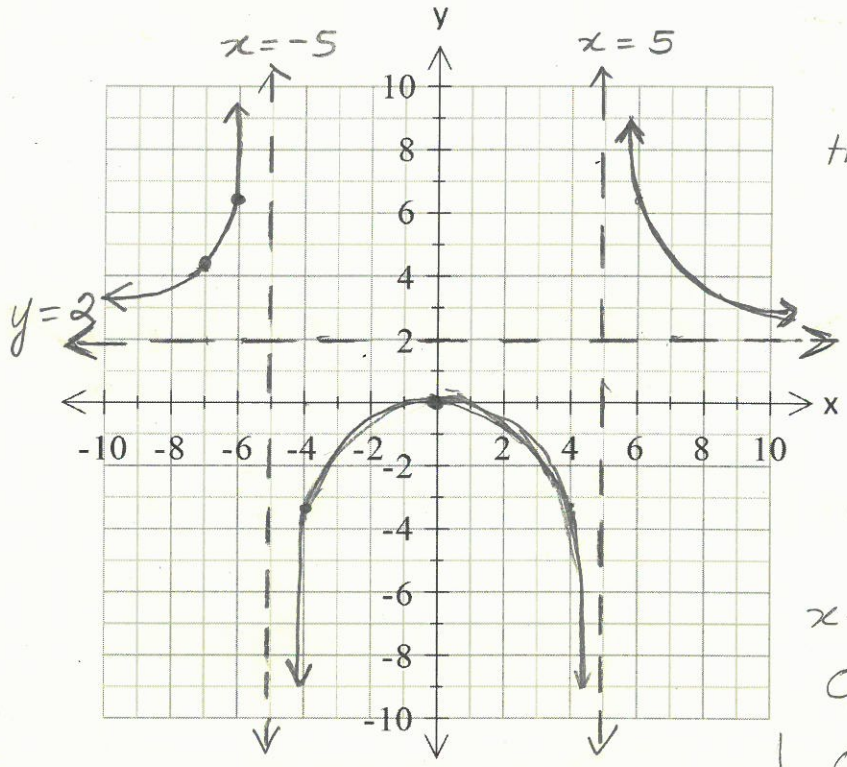
calculate x-int, set $y=0$

$0 = x - 3$

$x = 3$

Example 2: Sketch the graph of $y = \frac{2x^2}{x^2-25}$

$y = \frac{2x^2}{(x-5)(x+5)}$



deg $p(x)$ = deg $g(x)$

Horizontal asymptote $y = \frac{a}{b}$

$y = \frac{2}{1}$ $\{y = 2$

n.p.v. $x \neq 5$ $x \neq -5$

Vertical asymptotes are $x = 5$ and $x = -5$

NO Holes (no cancellation of common factors)

x-int, set $y = 0$

$0 = \frac{2x^2}{(x-5)(x+5)}$

$\frac{0}{2} = \frac{2x^2}{2}$

$\sqrt{0} = \sqrt{x^2}$

$0 = x$

y-int, set $x = 0$

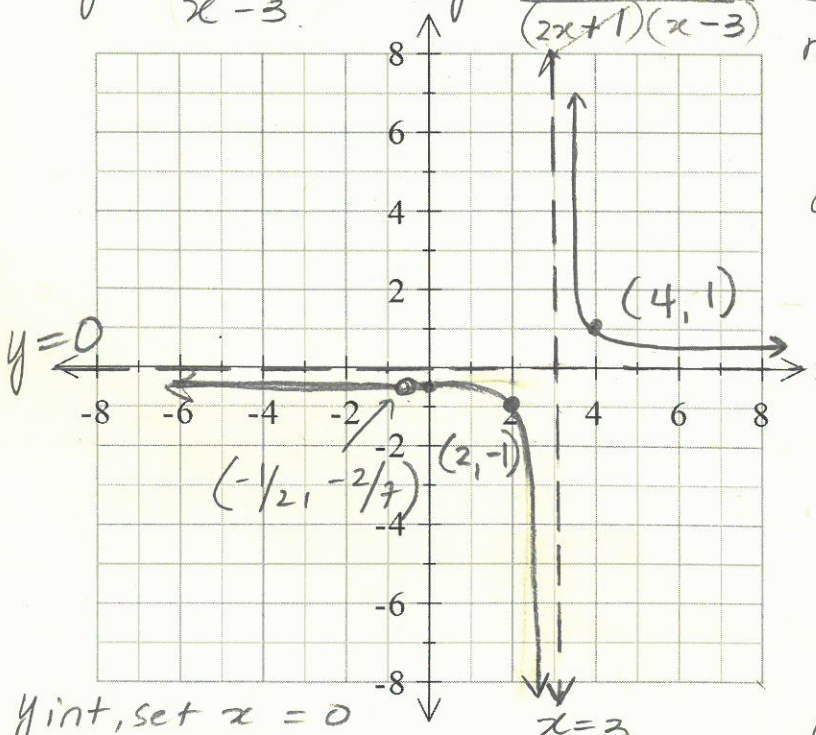
$y = \frac{2(0)^2}{0^2-25}$

$y = 0$

Example 3: Sketch the graph of $y = \frac{2x+1}{2x^2-5x-3}$

$y = \frac{1}{x-3}$

$y = \frac{(2x+1)\,1}{(2x+1)(x-3)}$



npv: $x \neq -\frac{1}{2}$ $x \neq 3$

Hole at $x = -\frac{1}{2}$

Coordinate of hole

$y = \frac{1}{x-3}$

$y = \frac{1}{-\frac{1}{2} - \frac{3}{1}}$

$y = \frac{1}{-\frac{7}{2}}$

$y = -\frac{2}{7}$

$(-\frac{1}{2}, -\frac{2}{7})$

Vertical asympt at $x = 3$

deg $p(x)$ < deg $g(x)$

$y = 0$

Horizontal Asymptote

y int, set $x = 0$

$y = \frac{1}{0-3}$

$y = -1/3$

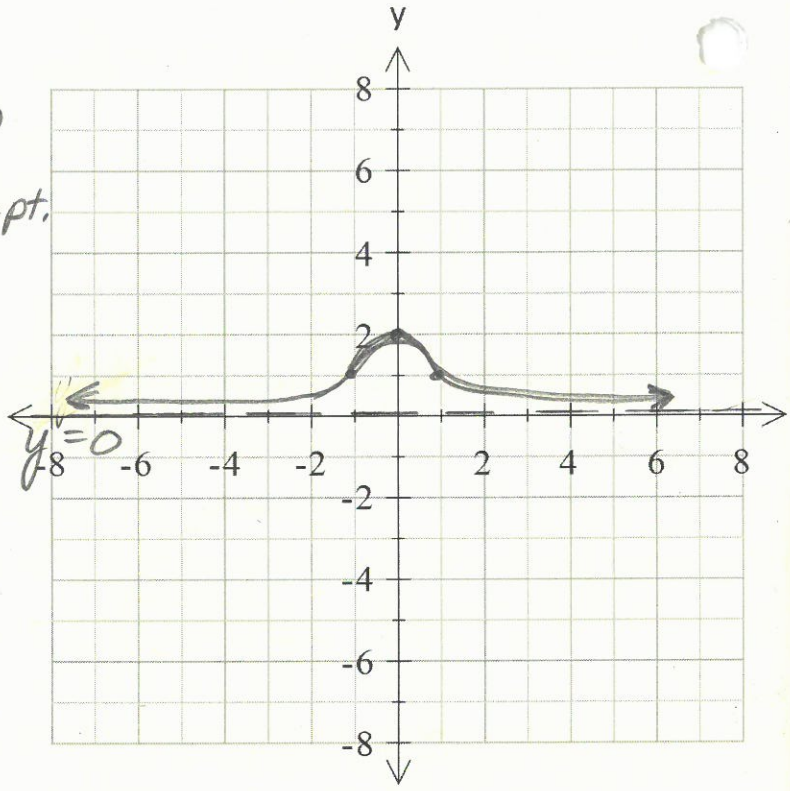
x int, set $y = 0$

$0 = \frac{1}{x-3}$

$0 = 1$ No soln

No x-int.

Example 4: Sketch the graph of $y = \frac{2}{x^2+1}$



npv $x^2+1 \neq 0$

$\sqrt{x^2} \neq \sqrt{-1}$

$x \neq \sqrt{-1}$

Therefore NO HOLE and Vertical Asympt.

deg $p(x)$ < deg $q(x)$

$y = 0$ is the Horizontal Asymptote.

y-int, set $x = 0$.

$y = \frac{2}{0^2+1}$

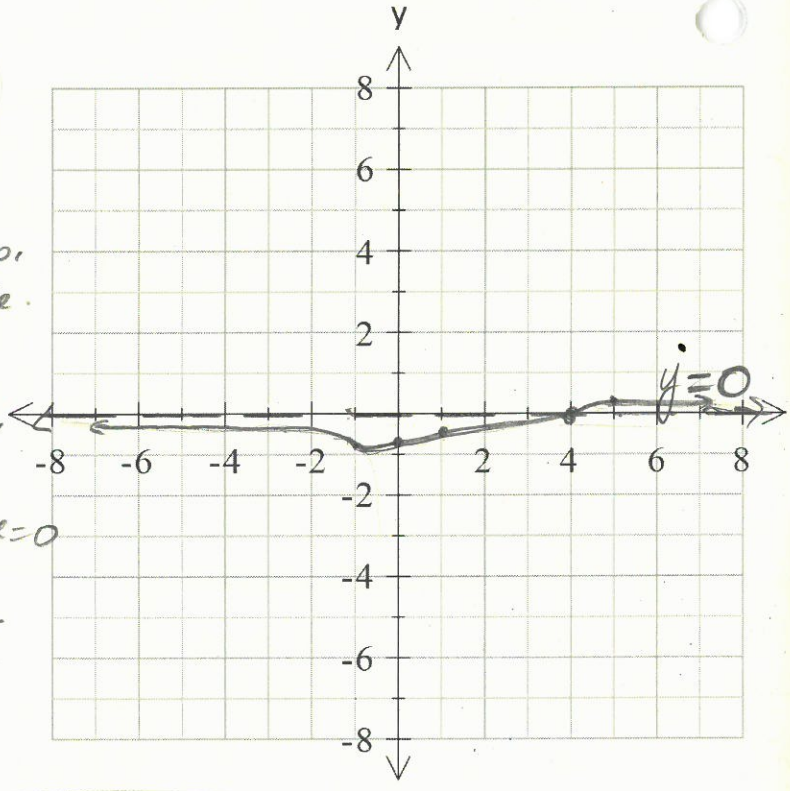
$y = 2$

x-int, set $y = 0$

$0 = \frac{2}{x^2+1}$

$0 = 2$ No x-int.

Example 5: Sketch the graph of $y = \frac{x-4}{x^2+5}$



npv. $x^2+5 \neq 0$

$\sqrt{x^2} \neq \sqrt{-5}$

$x \neq \sqrt{-5}$

Therefore no Vertical asymp. & and No Hole.

deg $p(x)$ < deg $q(x)$

$\therefore$ $y = 0$ Horizontal asymp.

x-int, set $y = 0$

$0 = \frac{x-4}{x^2+5}$

$0 = x - 4$

$x = 4$

y int, set $x = 0$

$y = \frac{0-4}{0^2+5}$

$y = -\frac{4}{5}$

$y = -0.8$

Assignment Time! Work on p.134- 3 – 5, 8, MC 1&2