Pre-Calculus 12: Solving Rational Equations

Solve the following rational equation algebraically. State any non-permissible values for x.

Example 1: $\frac{3}{x} = \frac{x-7}{6}$

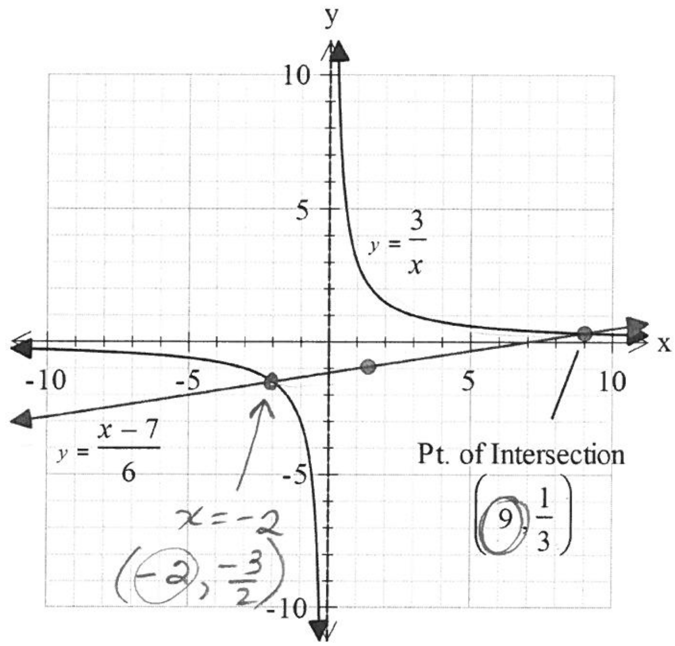
npv: $x \neq 0$
LCD $= 6x$

$$6x\left[\frac{3}{\cancel{x}}\right] = \left[\frac{x-7}{\cancel{6}}\right]\cancel{6x}$$

$$18 = (x-7)x$$
$$18 = x^2 - 7x$$
$$0 = x^2 - 7x - 18$$
$$0 = (x-9)(x+2)$$
$$x = 9 \qquad x = -2$$

Therefore the solutions are $x = 9$ and $x = -2$

The solution(s) can also be determined graphically. Identify on the graph shown below where the solutions from Example 1 are. Explain how the solutions from Example 1 correspond with the points you identified on the graph.



Example 2: $\frac{3}{2x} - \frac{2x}{x+1} = -2$

npv: $x \neq 0$ and $x = -1$
LCD: $2x(x+1)$

$$2x(x+1)\left[\frac{3}{\cancel{2x}} - \frac{2x}{x+1}\right] = [-2](2x)(x+1)$$

$$3(x+1) - 2x(2x) = -4x(x+1)$$
$$3x + 3 - \cancel{4x^2} = -\cancel{4x^2} - 4x$$
$$3x + 3 = -4x$$
$$3x + 4x = -3$$
$$\frac{7x}{7} = \frac{-3}{7}$$
$$\boxed{x = -\frac{3}{7}}$$

the solution is $x = -\frac{3}{7}$

Example 3: $\frac{x}{x-1} - 2x = \frac{x+1}{2x-2}$

npv: $x \neq 1$

LCD: $2(x-1)$

$$2(x-1)\left[\frac{x}{\cancel{x-1}} - 2x\right] = \left[\frac{x+1}{2(x-1)}\right](2)(x-1)$$

$$2x - 2x(2)(x-1) = x+1$$

$$2x - 4x(x-1) = x+1$$

$$2x - 4x^2 + 4x = x+1$$

$$0 = 4x^2 - 5x + 1$$

$$0 = (4x-1)(x-1)$$

$\boxed{x = \frac{1}{4}}$ $\quad$ $x = 1$

extraneous solution

Therefore the solution is $x = \frac{1}{4}$.

Example 4: $\frac{x-1}{x} = \frac{1}{x-1} - \frac{1}{x^2-x}$

$$x(x-1)\left[\frac{x-1}{x}\right] = \left[\frac{1}{x-1} - \frac{1}{x(x-1)}\right](x)(x-1)$$

npv: $x \neq 0$ and $x \neq 1$

LCD: $x(x-1)$

$$(x-1)(x-1) = x-1$$

$$x^2 - 2x + 1 = x - 1$$

$$x^2 - 2x + 1 - x + 1 = 0$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$x = 2$ $\quad$ $x = 1$

extraneous solution

therefore the solution is $\underline{\underline{x = 2}}$