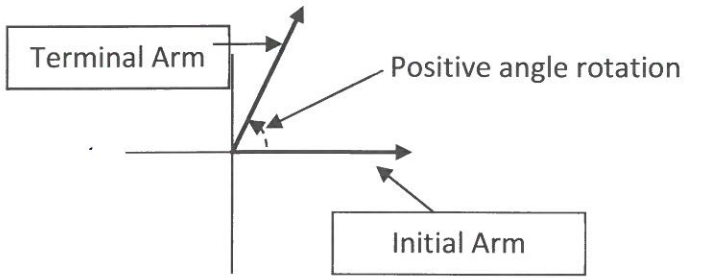
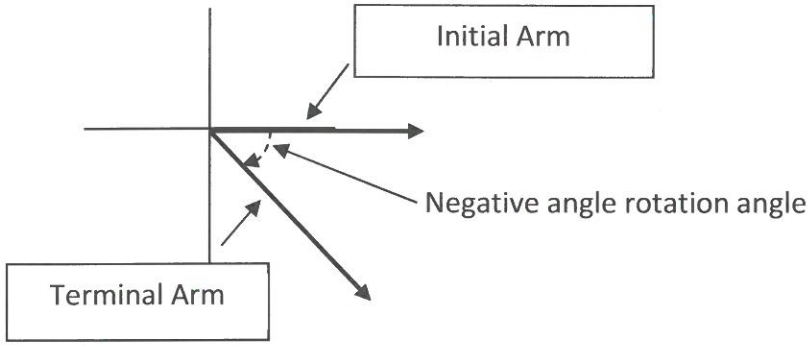
Lesson 1: Trigonometric Ratios for Any Angle in Standard Position

An angle is in standard position when its vertex is located at the origin [the point (0, 0)] and its initial arm lies on the x –axis. The terminal arm will be rotated in either a positive (counter-clockwise) or negative (clockwise) direction and stop at its terminal arm. Angles can be measured in degrees, radians, or rotations.

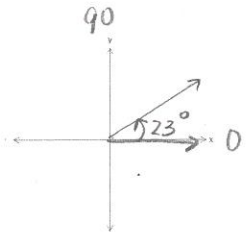




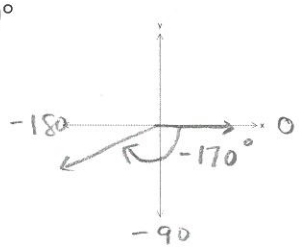
Example 1: Sketch the following angles in standard position:

a) 23°

Counter Clockwise



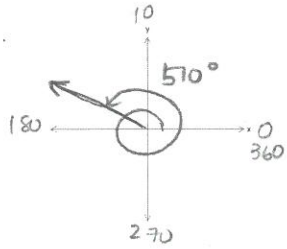
b) −170°



c) 510°

$$360\overline{)510}\quad 1\text{ R }150$$

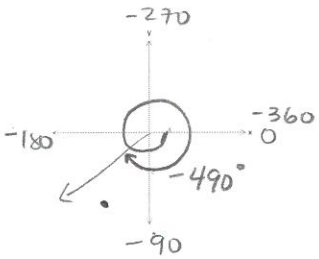
1 Full rotation and $\frac{150}{360}$°



d) −490°

$$360\overline{)490}\quad 1\text{ R }130$$

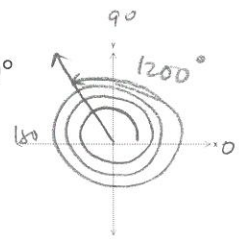
1 Full rotation and $\frac{130}{360}$



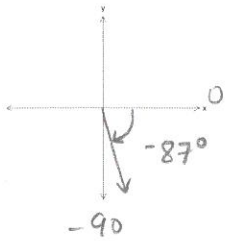
e) 1200°

$$360\overline{)1200}\quad 3\text{ R }120$$

3 full rotation and $\frac{120}{360}$



f) −87°



Coterminal Angles

Angles in standard position with the same terminal arm are coterminal angles. There are infinitely many angles that are coterminal with a given angle.

Given any angle θ, all of its coterminal angles can be defined as, in terms of θ,

$\theta + 360^0 k \quad where \quad k \in \mathbb{Z}$

set of integers

Example 2:

a) Determine the measures of all angles in standard position that are coterminal with an angle of 50°.

$360^\circ(k) + 50^\circ$

$k \in \mathbb{Z}$

$360(1) + 50^\circ = 410$

$360(2) + 50^\circ = 790$

$360(3) +$

$360(0) + 50 = 50$

$360(-1) + 50 = -310$

$360(-2) + 50 = -670^\circ$

b) Determine the measures of all angles between −720° and 720° that are coterminal with an angle of 20°.

$360(k) + 20^\circ$

$-700^\circ, -340^\circ, 20^\circ, 380^\circ,$

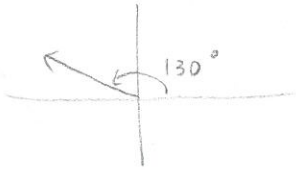
$360(-1) + 20 = -340$

$360(-2) + 20 = -700^\circ$

$360(1) + 20 = 380^\circ$

~~$360(2) + 20 = 740^\circ$~~ over 720°

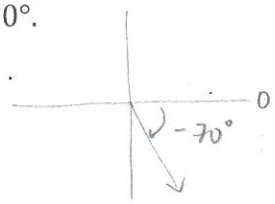
c) Determine one positive and one negative angle coterminal with an angle of 130°.



$360(1) + 130$
$= 360 + 130$
$= +490^\circ$

$360(-1) + 130$
$= -360 + 130$
$= -230^\circ$

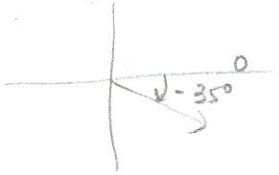
d) Determine one positive and one negative angle that are coterminal with an angle of −70°.



$360(1) + -70$
$= 360 + (-70)$
$= 290^\circ$

$360(-1) + (-70)$
$= -360 + (-70)$
$= -430^\circ$

e) Determine the measures of angles coterminal with the angle −35° for [−600°, 600°].



$360(-1) + (-35)$
$= -395^\circ$

$360(0) + (-35)$
$= -35$

$360(1) + (-35^\circ)$
$= 325^\circ$

$-395^\circ, -35^\circ, 325^\circ$