Lesson 3: Introduction to Composite Functions

Example 1: The tables below define two functions. Use these tables to determine the values requested below the tables.

Work from inside to outside.

x	$f(x)$
-2	8
-1	3
0	0
1	-1
2	0

x	$g(x)$
-2	3
-1	2
0	1
1	0
2	1

a) $f(g(2))$

$= f(1)$

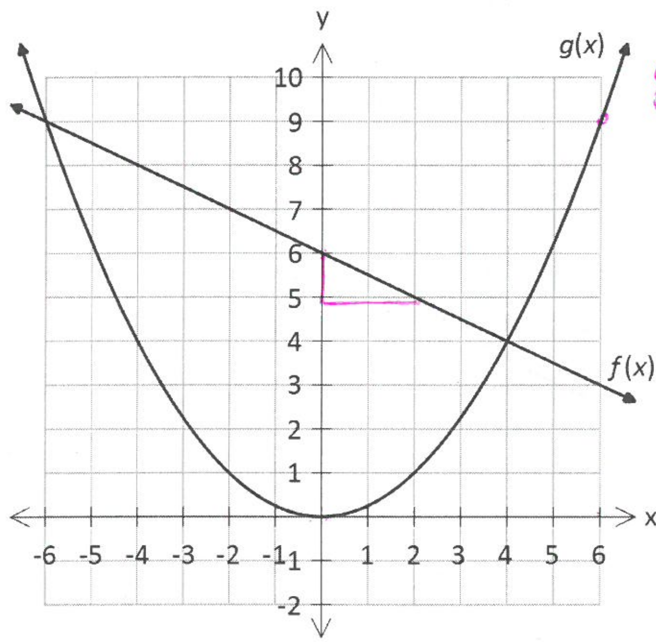
$= -1$

b) $g(g(-1))$

$= g(2)$

$= 1$

Example 2: Given the graphs of $y = f(x)$ and $y = g(x)$, determine the values requested below the graphs.



$g(x) = \left(\frac{1}{2}x\right)^2$

$f(x) = -\frac{1}{2}x + 6$

a) $g(f(0))$

$= g(6)$

$= 9$

b) $f(g(1))$

$g(1) = \left(\frac{1}{2}(1)\right)^2$

$g(1) = \frac{1}{4}$

$f(g(1))$

$= f\left(\frac{1}{4}\right)$

$= -\frac{1}{2}\left(\frac{1}{4}\right) + 6$

$= -\frac{1}{8} + 6$

$= 5\frac{7}{8}$

Example 3: Given the functions $h(x) = \sqrt{x+5}$ and $m(x) = (x-1)^2$, determine the values requested below:

a) $m(h(4))$

$h(4) = \sqrt{4+5}$
$h(4) = \sqrt{9}$
$h(4) = 3$

$m(h(4))$
$= m(3)$
$= (3-1)^2$
$= (2)^2$
$= \boxed{4}$

b) $h(m(13))$

$m(13) = (13-1)^2$
$m(13) = (12)^2$
$m(13) = 144$

$h(m(13))$
$= h(144)$
$= \sqrt{144+5}$
$= \sqrt{149}$

Example 4: Given $f(x) = x^2 + 3x$ and $g(x) = 3x - 5$, determine an explicit equation for each requested composite function, and state the domain of each composite function. (**OPTIONAL** Use graphing technology to graph each composite function and determine the range.)

a) $f(g(x))$

b) $g(f(x))$

c) $f(f(x))$

* Solutions and graphs on separate pages.

Assignment Time! Work on p.298- 4 – 11, MC 1&2

Pg 10) Example 4

$f(x) = x^2 + 3x$ $\qquad$ $g(x) = 3x - 5$

a) $f(g(x)) = f(3x-5)$

$= (3x-5)^2 + 3(3x-5)$

$= 9x^2 - 30x + 25 + 9x - 15$

$= 9x^2 - 21x + 10$

$$\boxed{f(g(x)) = 9x^2 - 21x + 10}$$

b) $g(f(x)) = g(x^2+3x)$

$= 3(x^2+3x) - 5$

$= 3x^2 + 9x - 5$

$$\boxed{g(f(x)) = 3x^2 + 9x - 5}$$ ✓

c) $f(f(x)) = f(x^2+3x)$

$= (x^2+3x)^2 + 3(x^2+3x)$

$= x^4 + 6x^3 + 9x^2 + 3x^2 + 9x$

$= x^4 + 6x^3 + 12x^2 + 9x$

$$\boxed{f(f(x)) = x(x^3 + 6x^2 + 12x + 9)}$$

$= (x)(x+3)(x^2+3x+3)$

Example #4

GRAPH

a) $f(g(x)) = 9x^2 - 21x + 10$

$D: x \in \mathbb{R}$

$R: [-2.25, \infty)$

Determine the vertex: $y = a(x-p)^2 + q$

$$p = \frac{-b}{2a}$$

$$p = \frac{-(-21)}{2(9)}$$

$$p = \frac{21}{18} \div \frac{3}{3}$$

$$p = \frac{7}{6}$$ ← x-value of vertex.

$$y = 9\left(\frac{7}{6}\right)^2 - 21\left(\frac{7}{6}\right) + 10$$

$$y = \cancel{9}^{1}\left(\frac{49}{\cancel{36}_{4}}\right) - \cancel{21}^{7}\left(\frac{7}{\cancel{6}_{2}}\right) + (10)$$

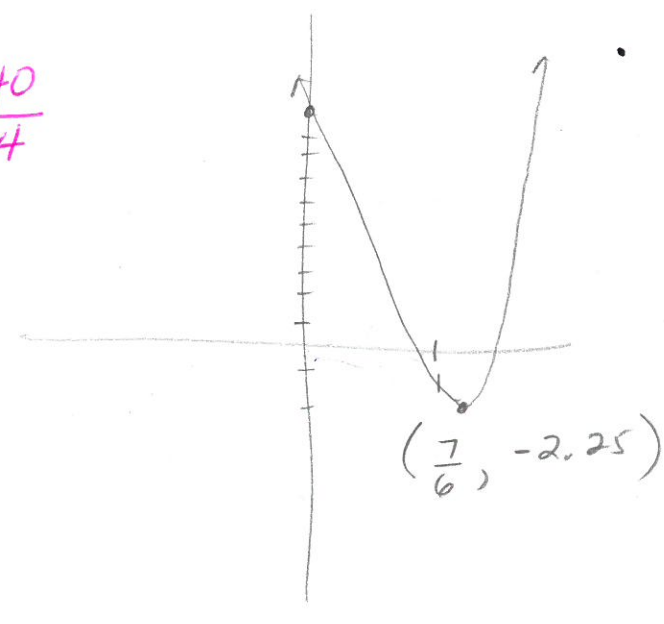
$$y = \frac{49}{4} - \frac{49}{2} + 10$$

$$y = \frac{49}{4} - \frac{98}{4} + \frac{40}{4}$$

$$y = \frac{-98}{4} + \frac{89}{4}$$

$$y = \frac{-9}{4}$$

$$y = -2.25$$



GRAPH

b) $g(f(x)) = 3x^2 + 9x - 5$

D: $x \in \mathbb{R}$.

R: $y \geq -11.75$

$p = \frac{-b}{2a}$

$p = \frac{-9}{2(3)}$

$p = \frac{-9}{6}$

$p = -\frac{3}{2}$ ← x-coord of vertex.

$y = 3\left(-\frac{3}{2}\right)^2 + 9\left(-\frac{3}{2}\right) - 5$

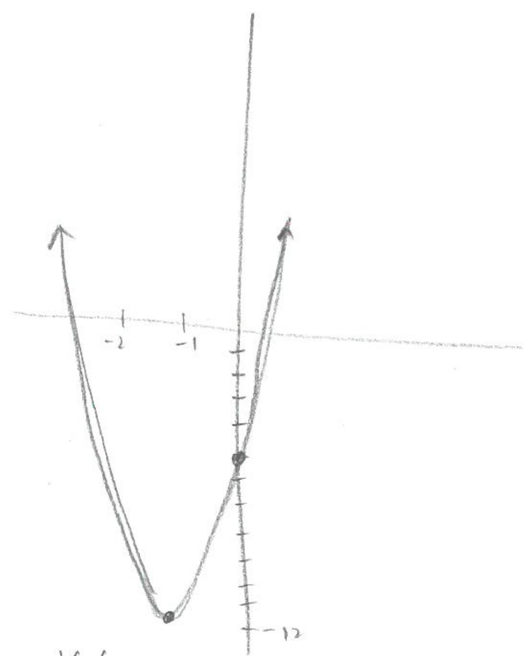
$y = -11.75$ Vertex $\left(-\frac{3}{2}, -11.75\right)$

$y = 3\left(\frac{9}{4}\right) + -\frac{27}{2} - 5$

$y = \frac{27}{4} - \frac{27}{2} - 5$

$y = \frac{27}{4} - \frac{54}{4} - \frac{20}{4}$

$y = -11.75$



V $\left(-\frac{3}{2}, -11.75\right)$