Lesson 4: Determining Restrictions on Composite Functions

Example 1: Use the functions $f(x) = 2x - 1$ and $g(x) = x^2 - 2$.

a) State the domain of $f(x)$ and of $g(x)$. $f(x)$ D: $x \in \mathbb{R}$ R: $y \in \mathbb{R}$ } $g(x)$ D: $x \in \mathbb{R}$ R: $y \geq -2$

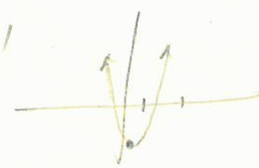
b) Use graphing technology to sketch a graph of $y = g(f(x))$ and determine the domain of this composite function.

c) Use graphing technology to sketch a graph of $y = g(g(x))$ and determine the domain of this composite function.

b) $g(f(x))$
$= g(2x-1)$
$= (2x-1)^2 - 2$
$= 4x^2 - 4x + 1 - 2$
$= 4x^2 - 4x - 1$

Vertex
$p = \frac{-b}{2a}$
$p = \frac{-(-4)}{2(4)}$
$p = \frac{4}{8}$
$p = 1/2$

$g = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) - 1$
$g = 4\left(\frac{1}{4}\right) - 2 - 1$
$g = 1 - 2 - 1$
$g = -2$



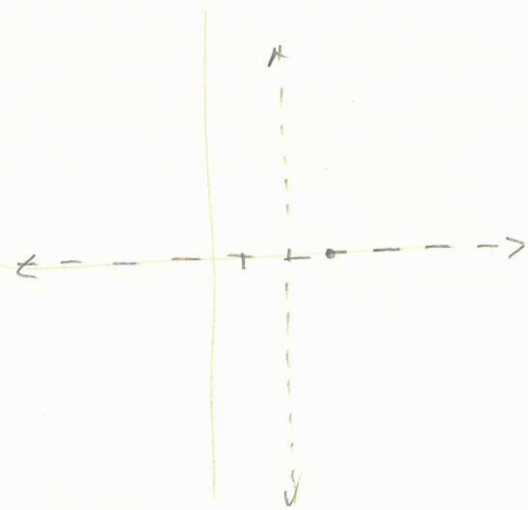
D: $x \in \mathbb{R}$
R: $[-2, \infty)$

Example 2: Given the functions $h(x) = \frac{1}{x-2}$ and $j(x) = x^2 - x$, determine an explicit equation of each composite function below, then state its domain.

a) $j(h(x))$ b) $h(j(x))$

OPTIONAL Verify your answers using graphing technology.

a) $j(h(x))$
$= j\left(\frac{1}{x-2}\right)$
$= \left(\frac{1}{x-2}\right)^2 - \left(\frac{1}{x-2}\right)$
$= \frac{1}{x^2-2x+4} - \frac{1}{x-2}$
$= \frac{1}{(x-2)^2} - \frac{1}{(x-2)} \cdot \frac{(x-2)}{(x-2)}$
$= \frac{1}{(x-2)^2} - \frac{(x-2)}{(x-2)^2}$
$= \frac{1-x+2}{(x-2)^2}$
$= \frac{-x+3}{(x-2)^2} = \frac{-1(x-3)}{(x-2)^2}$



npv: $x \neq 2$
vertical asympt $x = 2$

$y = \frac{-1(x-3)}{(x-2)^2}$

y-int, set $x = 0$
$y = \frac{-1(-3)}{(-2)^2}$ $\boxed{y = \frac{3}{4}}$

x-int, set $y = 0$
$0 = \frac{-1(x-3)}{(x-2)^2}$
$x - 3 = 0$
$\boxed{x = 3}$

Example #2 9) $h(x) = \dfrac{1}{x-2}$ $j(x) = x^2 - x$

a) $j(h(x))$

$= j\left(\dfrac{1}{x-2}\right)$

$= \left(\dfrac{1}{x-2}\right)^2 - \left(\dfrac{1}{x-2}\right)$

$= \dfrac{1}{(x-2)^2} - \dfrac{1}{(x-2)}$

$= \dfrac{1}{(x-2)^2} - \dfrac{1(x-2)}{(x-2)(x-2)}$

$= \dfrac{1}{(x-2)^2} - \dfrac{(x-2)}{(x-2)^2}$

$= \dfrac{1 - x + 2}{(x-2)^2}$

$= \dfrac{-x+3}{(x-2)^2}$

$\boxed{y = \dfrac{-x+3}{(x-2)^2}}$ ← EXPLICIT EQ'N.

---

npv: $x - 2 \neq 0$

$x \neq 2$

VA: $x = 2$

HA: $y = 0$

y-int, set $x = 0$

$y = \dfrac{-0+3}{(0-2)^2}$

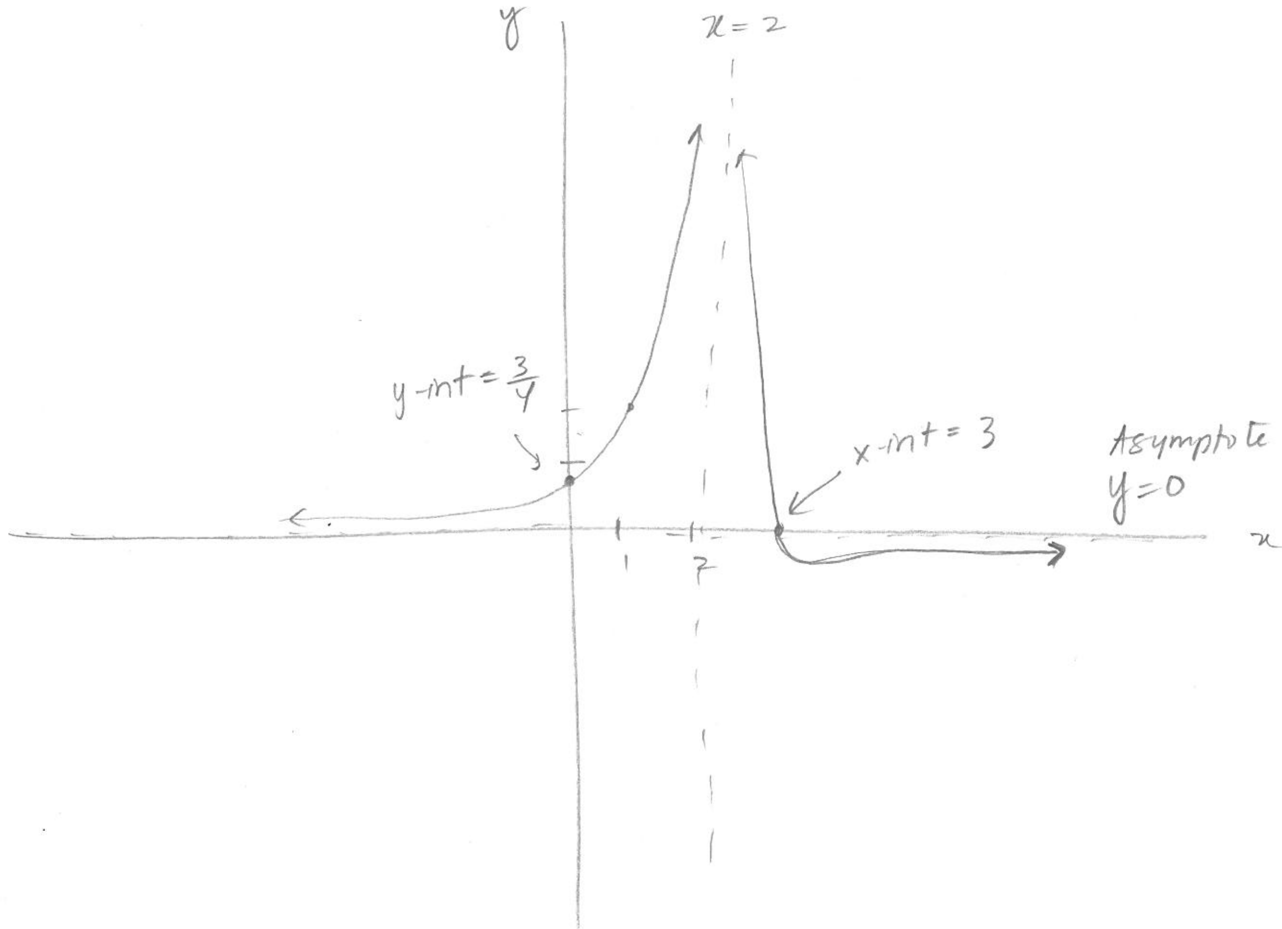
$\boxed{y = \dfrac{3}{4}}$

x-int, set $y = 0$

$0 = \dfrac{-x+3}{(x-2)^2}$

$0 = -x + 3$

$\boxed{x = 3}$



Test point $x = 1$

$$y = \frac{-1+3}{(1-2)^2}$$

$$y = \frac{2}{1}$$

$$y = 2$$

Test point. $x = 3$

$$y = \frac{-3+3}{}$$

Domain: $x \in \mathbb{R},\ x \neq 2$

RANGE:

We might need desmos to get the range, at least just the minimum value for y.

Example #2b $h(x) = \frac{1}{x-2}$ $\quad j(x) = x^2 - x$

$h(j(x))$

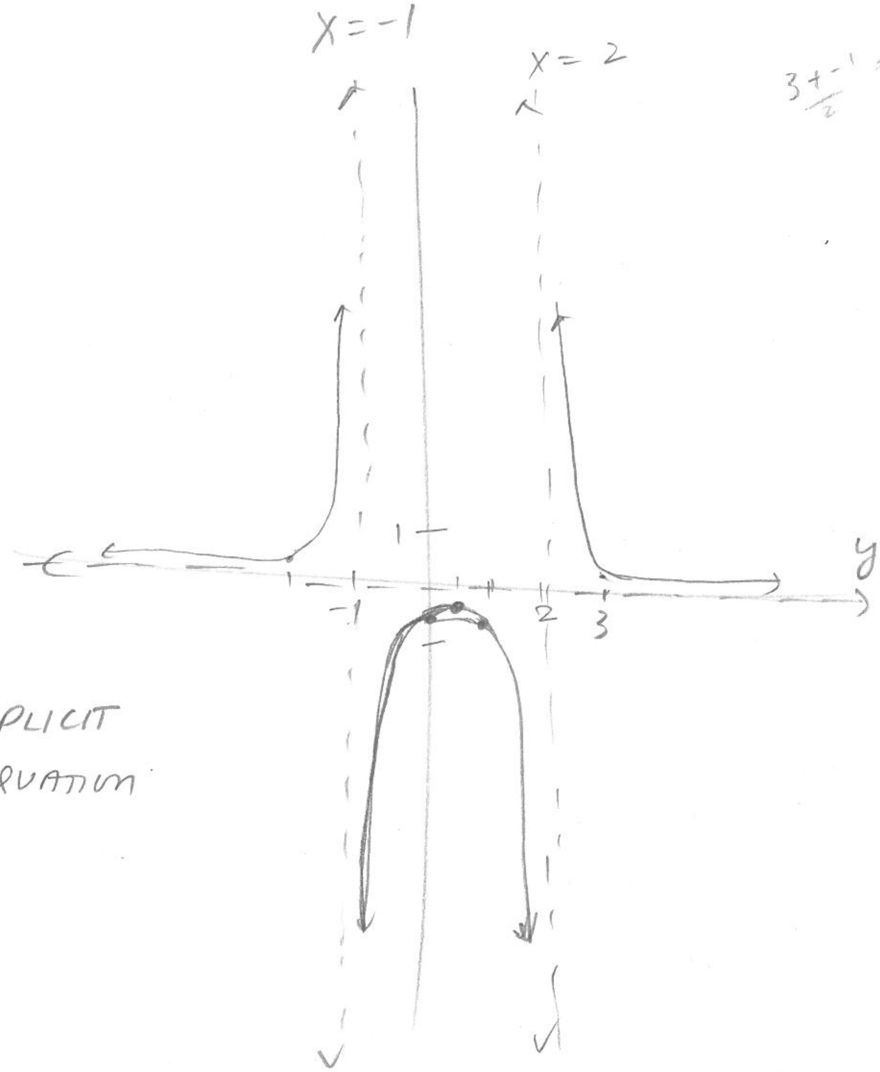
$= h(x^2 - x)$

$= \frac{1}{(x^2-x)-2}$

$= \frac{1}{x^2-x-2}$

$= \frac{1}{(x-2)(x+1)}$

$\boxed{y = \frac{1}{(x-2)(x+1)}}$ ← EXPLICIT EQUATION



npv: $x \neq 2, -1$

VA: $x = 2$, $x = -1$

HA: $y = 0$

x-int, set $y = 0$

$0 = \frac{1}{(x-2)(x+1)}$

$0 = 1$

No solution. Therefore no x-int

y-int, set $x = 0$

$y = \frac{1}{(-2)(+1)}$

$\boxed{y = \frac{1}{-2}}$

Test point $x = 3$

$y = \frac{1}{(3-2)(3+1)}$

$y = \frac{1}{4}$

Test point $x = -2$

$y = \frac{1}{(-2-2)(-2+1)}$

$y = \frac{1}{4}$

Test pt. $x = 1$

$y = \frac{1}{(1-2)(1+1)}$

$y = \frac{1}{-2}$

$x = 0.5$

$y = \frac{1}{(\frac{1}{2} - 2)(\frac{1}{2} + 1)}$

$y = \frac{1}{(-\frac{3}{2})(\frac{3}{2})} = \frac{1}{-\frac{9}{4}}$

D: $x \in \mathbb{R}$, $x \neq -1, +2$

R: $(-\infty, -4/9) \cup (0, \infty)$

Example 3) $f(x) = \sqrt{x}$ $\quad g(x) = -x^2 + 2x$

a) $g(f(x))$

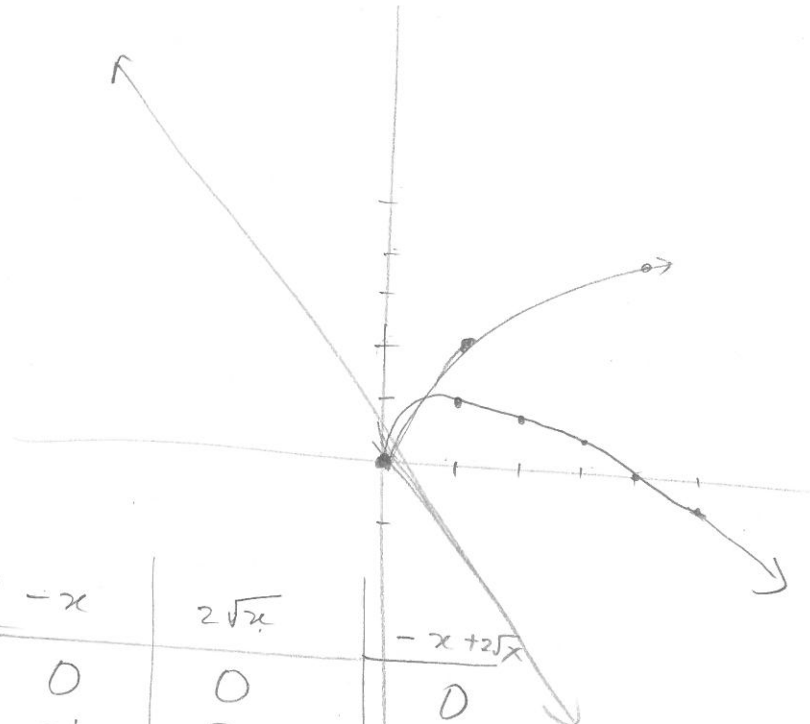
$= g(\sqrt{x})$

$= -(\sqrt{x})^2 + 2\sqrt{x}$

$= -x + 2\sqrt{x}$

WE KNOW HOW TO DO THIS!

from lesson 1.



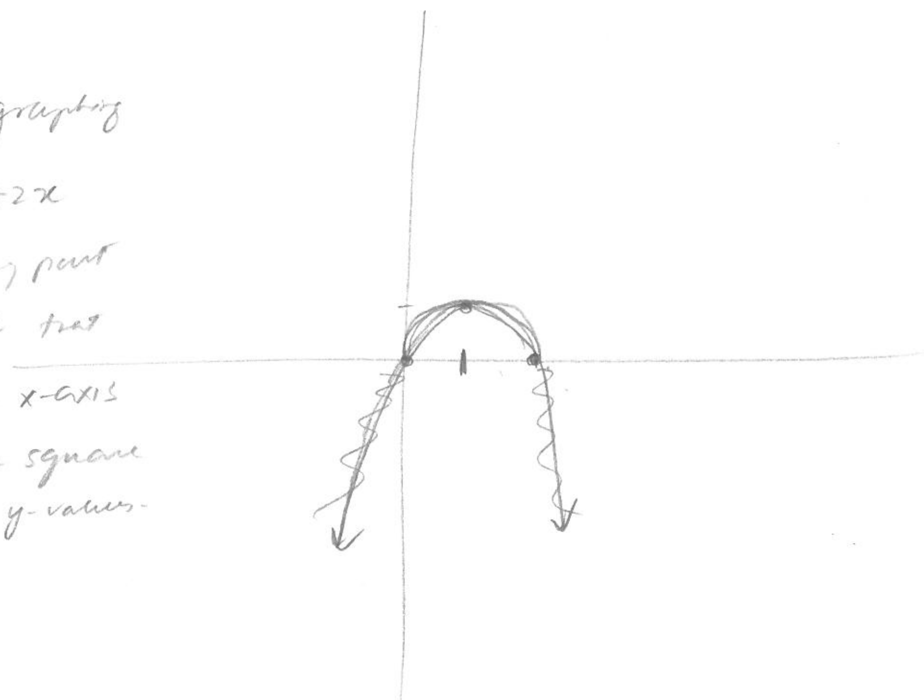
x	$-x$	$2\sqrt{x}$	$-x + 2\sqrt{x}$
0	0	0	0
1	-1	2	1
2	-2	2.83	0.83
3	-3	3.46	0.46
4	-4	4	0
5	-5	4.47	-0.53

Domain of $y = -x + 2\sqrt{x}$ is $[0, \infty)$

Example #3 b) $f(x) = \sqrt{x}$ $g(x) = -x^2 + 2x$

$$y = f(g(x))$$
$$= f(-x^2+2x)$$
$$= \sqrt{-x^2+2x}$$ ← WE KNOW HOW TO DO THIS from CHAPTER 2
$$= \sqrt{-x(x-2)}$$

Start w/ graphing $y = -x^2 + 2x$ take out any part of the graph that is below the x-axis then take the square root of the y-values.



Example # 4

a) $y = f(g(x))$

$y = \frac{1}{\sqrt{x}}$ $\quad g(x) = \sqrt{x}$ $\quad f(x) = \frac{1}{x}$

b) $y = |2x-1|^5$ $\quad g(x) = |2x-1|$ $\quad f(x) = x^5$