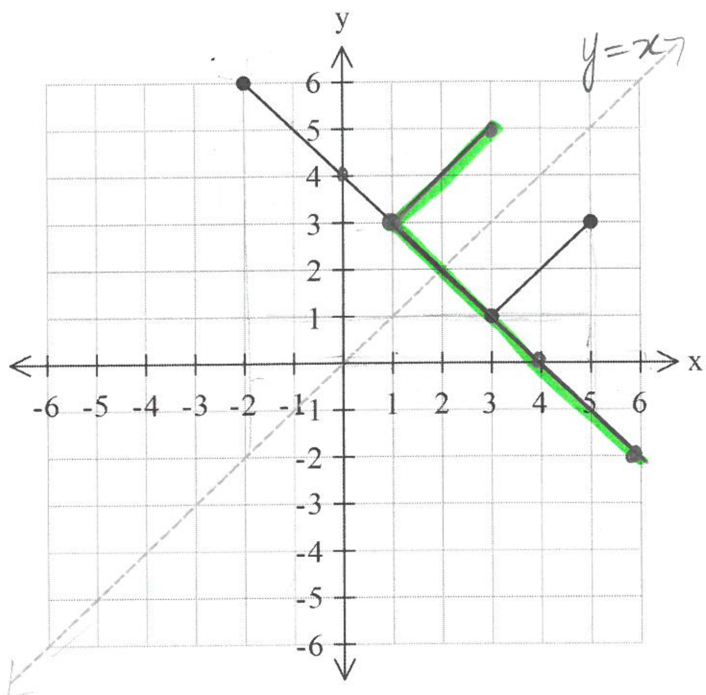
Lesson 5: Inverses of Relations

Functions that are reflections of each other in the line $y = x$ are called inverses of each other. A reflection the in the line $y = x$ results in the x and y values "switching" places. The inverse of a function $f(x)$ is denoted by the notation $f^{-1}(x)$.

Example 1:
Graph the inverse of the function $f(x)$ shown below.



$f(x)$	$f^{-1}(x)$
(x, y)	(y, x)
$(-2, 6)$	$(6, -2)$
$(3, 1)$	$(1, 3)$
$(5, 3)$	$(3, 5)$
$(0, 4)$	$(4, 0)$

Domain and Range of $f(x)$:

D: $[-2, 5]$

R: $[1, 6]$

Domain and Range of $f^{-1}(x)$.

D: $[1, 6]$

R: $[-2, 5]$

Example 2:

Algebraically determine the equation of the inverse of each function. Sketch graphs of $f(x)$ and its inverse.

a) $f(x) = \frac{1}{2}x + 1$

$y = \frac{1}{2}x + 1$

- interchange variables x and y

$x = \frac{1}{2}y + 1$

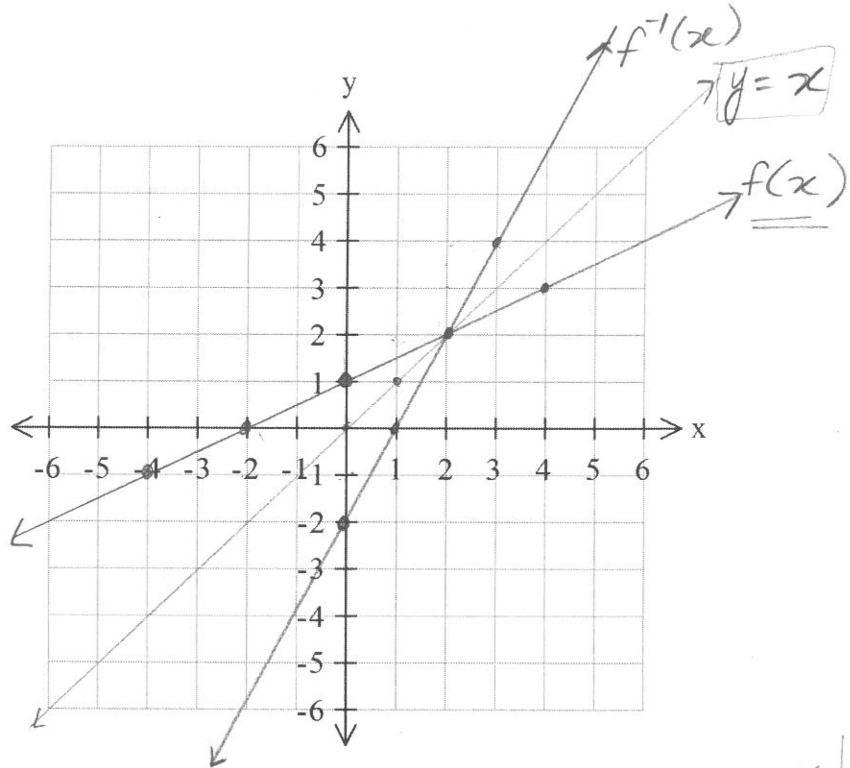
- Solve for y-variable

$x - 1 = \frac{1}{2}y$

$2(x-1) = y$

$2x - 2 = y$

$\boxed{f^{-1}(x) = 2x - 2} \rightarrow \boxed{y = 2x - 2}$



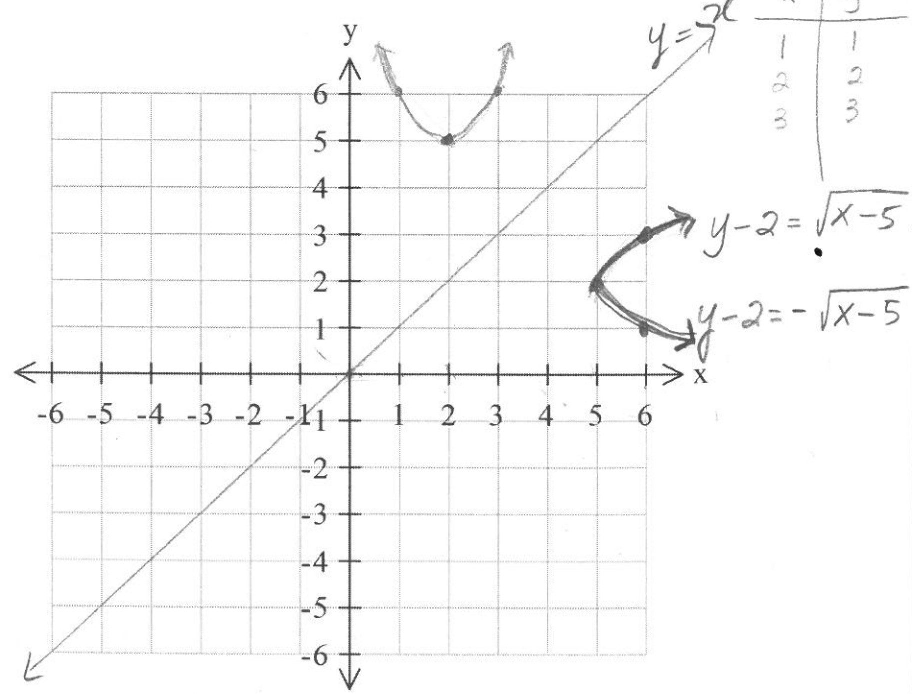
b) $y = (x-2)^2 + 5$

parent function

$y = x^2$

(x, y)	$(x+2, y+5)$
$(0,0)$	$(2,5)$
$(1,1)$	$(3, 6)$
$(2,4)$	$(4, 9)$
$(-1,1)$	$(1, 6)$
$(-2,4)$	$0, 9)$

$f(x)$ (x, y)	$f^{-1}(x)$ (y, x)
$(1,6)$	$(6,1)$
$(2,5)$	$(5,2)$
$(3,6)$	$(6,3)$



x	y
1	1
2	2
3	3

Example 3:

a) Determine an equation of the inverse of $y = (x-1)^2 + 2$

b) Sketch graph of the function and its inverse.

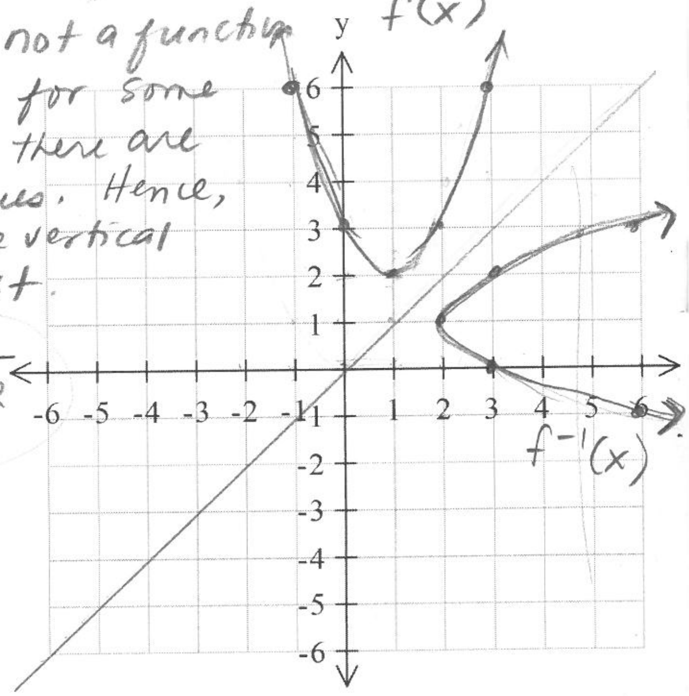
c) Is the inverse a function? Explain. Relation, not a function because for some x-values there are 2 y-values. Hence, it fails the vertical line test.

a) $x = (y-1)^2 + 2$

$\pm\sqrt{x-2} = \sqrt{(y-1)^2}$

$\pm\sqrt{x-2} = y-1$

$y-1 = \sqrt{x-2}$ AND $y-1 = -\sqrt{x-2}$



(x, y)	(y, x)
$(-1, 6)$	$(6, -1)$
$(0, 3)$	$(3, 0)$
$(1, 2)$	$(2, 1)$
$(2, 3)$	$(3, 2)$
$(3, 6)$	$(6, 3)$

Example 4:

a) Determine algebraically the inverse of $y = -x^2 + 6$

b) Restrict the domain of the base function so that the inverse is a function.

c) Verify by sketching the graph of the base function and its inverse.

$y = -x^2 + 6$

$x = -y^2 + 6$

$\dfrac{x-6}{-1} = \dfrac{-1y^2}{-1}$

$\sqrt{-x+6} = \sqrt{y^2}$

$\pm\sqrt{-x+6} = y$

OMIT

$y = \sqrt{-x+6}$ AND $y = -\sqrt{-x+6}$

D: $-x+6 \geq 0$

$6 \geq x$

$x \leq 6$