Problems involving Cases

Sometimes a permutation problem involves more than one restriction and the number of choices for one restriction affects the number of choices in another restriction. In this situation, you must split the question into separate ***cases***.

Example 1: How many four-letter arrangements beginning with either B or E and ending with a vowel can you make using the letters A, B, C, E, U, and G?

$$\underset{B}{\underline{1}} \cdot \underline{4} \cdot \underline{3} \cdot \underset{\text{vowel}}{\underline{3}} = 36$$

$$\underset{E}{\underline{1}} \cdot \underline{4} \cdot \underline{3} \cdot \underset{\text{vowel}}{\underline{2}} = 24$$

Total 4-letter arrangement is $36 + 24 = 60$ ways.

Example 2: How many 3-digit even numbers greater than 200 can you make using the digits 1, 2, 3, 4, and 5?

must start w/ either 2, 3, 4, 5

CASE 1: start w/ 2 $\quad \underset{\text{start w/ 2}}{\underline{1}} \cdot \underline{3} \cdot \underset{\text{end w/ } \cancel{2} \text{ or } 4}{\underline{1}} = 3$

CASE 2: start w/ 3 $\quad \underset{3}{\underline{1}} \; \underline{3} \; \underset{\text{end w 2 or 4}}{\underline{2}} = 6$

CASE 3: start w/ 4 $\quad \underset{4}{\underline{1}} \cdot \underline{3} \cdot \underset{\text{end w/ 2 or } \cancel{4}}{\underline{1}} = 3$

CASE 4: start w/ 5 $\quad \underset{5}{\underline{1}} \cdot \underline{3} \cdot \underset{\text{end w/ 2 or 4}}{\underline{2}} = 6$

Total ways: $3 + 6 + 3 + 6 = \boxed{18 \text{ ways}}$

Example 3: How many ways can four girls and three boys be arranged in a row if the ends of the row must be either both boys or both girls?

CASE 1: Both boys

$$\underset{\text{Boy}}{\underline{3}} \cdot \underbrace{\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}}_{\text{rest of the people}} \cdot \underset{\text{Boy}}{\underline{2}} = 720 \text{ ways}$$

CASE 2: Both girls

$$\underset{\text{girl}}{\underline{4}} \; \underline{5} \; \underline{4} \; \underline{3} \; \underline{2} \; \underline{1} \; \underset{\text{girl}}{\underline{3}} = 1440 \text{ ways}$$

total ways $= 720 + 1440 = 2160$ ways

Permutation order matters $_nP_r = \frac{n!}{(n-r)!}$

Combination order doesn't matter $_nC_r = \frac{n!}{(n-r)!\,r!} = \binom{n}{r}$ ← books

Lesson 5: Combinations

While a permutation is a selection and *ordering* of a group of objects, a combination is a selection of a group of objects where the order of the objects is *not* important. In other words, a combination is a permutation without regard to arranging the objects.

We can calculate the number of combinations of a selection of objects by using reasoning or by using the formula $_nC_r = \frac{n!}{(n-r)!r!}$. There is also a combination button on your calculator.

Example 1: Use the formula to calculate:

a) $_{10}C_4$ $= \frac{10!}{(10-4)!(4)!} = \frac{10\cdot\cancel{9}\cdot\cancel{8}\cdot 7\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 210$ ways

b) $_{10}C_6$ $= \frac{10!}{(10-6)!\,6!} = \frac{10\cdot\cancel{9}\cdot\cancel{8}\cdot 7\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 210$ ways

Comment on the answers to a) and b) above.

c) Explain why $_4C_7$ isn't possible. cannot select 7 objects from only 4 objects to begin with.

start w/ 4 objects

select 7

Example 2: How many committees of three people can be selected from a class of 20 students?

selecting 3 order does not matter.

$_{20}C_3 = \binom{20}{3} = 1140$ ways

Example 3: To play the Super 7 lottery, you must choose seven numbers from 1 to 47. To play in the Lotto 649, you must choose six numbers from 1 to 49. To win each jackpot, the numbers chosen must match the numbers drawn by the lottery corporation. Determine the number of possible winning combinations there are for each jackpot.

Super 7: $_{47}C_7 = \binom{47}{7} = 62,891,499$

Lotto 649: $_{49}C_6 = \binom{49}{6} = 13,983,816$

Example 4: There is a group of people consisting of 15 men and 12 women.

a) Determine the number of 5-person committees that can be formed from this group of people.

$_{27}C_5 = \binom{27}{5} = 80,730$ ways

b) Determine the number of 5-person committees that can be formed if there must be 3 men and 2 women on the committee.

$_{15}C_3 \times {}_{12}C_2$

$455 \times 66 = 30,030$ ways

c) Determine the number of 5-person committees that can be formed if there must be a **majority** of women on the committee.

CASE 1: 2M & 3W $= {}_{15}C_2 \times {}_{12}C_3 = 23,100$

CASE 2: 1M & 4W $= {}_{15}C_1 \times {}_{12}C_4 = 7425$

CASE 3: 0M & 5W $= {}_{15}C_0 \times {}_{12}C_5 = 792$

31,317 ways

d) Peta is one of the women in this group. Determine the number of 5-person committees that can be formed if Peta must be on the committee.

Peta and 4 more people

$_1C_1 \times {}_{26}C_4 = 14,950$ ways

Assignment Time! $\boxed{\text{Work on p.727- 4 – 15 (not 6), MC 1-3}}$