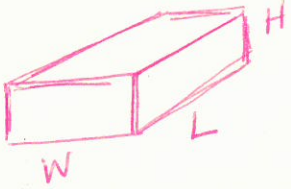
Lesson 6: Modelling and Solving Problems with Polynomial Functions

Example 1: The volume, in cubic centimetres, of a rectangular box can be modeled by the polynomial function $V(x) = 3x^3 + x^2 - 12x - 4$. Determine expressions for the other dimensions of the box if the height is $x + 2$.



$V = \ell \times w \times h$

$V = 3x^3 + x^2 - 12x - 4$

$$\begin{array}{r|cccc} -2 & 3 & 1 & -12 & -4 \\ & & -6 & +10 & 4 \\ \hline & 3 & -5 & -2 & 0 \end{array}$$

$3x^2 - 5x - 2$

$3x^2 - 6x + x - 2$

$3x(x-2) + 1(x-2)$

$(x-2)(3x+1)$

Therefore the other dimensions are $(x-2)(3x+1)$

Example 2: An artist creates a carving from a block of soapstone. The soapstone is in the shape of a rectangular prism (box) whose volume, in cubic feet, is represented by the equation $V(x) = 6x^3 + 25x^2 + 2x - 8$, where x is a positive real number. What are the factors that represent possible dimensions, in terms of x, of the block of soapstone?

$V(-1) = -6 + 25 - 2 - 8$

$V(-1) = 9$ $(x+1)$ not a factor

$V(-2) = 6(-8) + 25(4) + 2(-2) - 8$

$V(-2) = -48 + 100 + 4 - 8$

$V(-4) = 6(-4)^3 + 25(-4)^2 + 2(-4) - 8$

$= -384 + 400 - 8 - 8$

$= 0$ $(x+4)$ is a factor

$$\begin{array}{r|cccc} -4 & 6 & 25 & 2 & -8 \\ & & -24 & -4 & 8 \\ \hline & 6 & 1 & -2 & 0 \end{array}$$

$6x^2 + x - 2$

$6x^2 + 4x - 3x - 2$

$2x(3x+2) - 1(3x+2)$

$(3x+2)(2x-1)$

Therefore

$V(x) = (3x+2)(2x-1)(x+4)$

Example 3: Three consecutive odd integers have a product of −105. What are the three integers?

1st Odd $2x+1$ 2nd odd $2x+3$ 3rd odd $2x+5$

$(2x+1)(2x+3)(2x+5) = -105$

$(4x^2+6x+2x+3)(2x+5) = -105$

$(4x^2+8x+3)(2x+5) = -105$

$8x^3+20x^2+16x^2+40x+6x+15 = -105$

$8x^3+36x^2+46x+15+105 = 0$

$8x^3+36x^2+46x+120 = 0$

$2(4x^3+18x^2+23x+60) = 0$

$P(-2) = 54$

$P(-3) = 45$

$P(-4) = 0$ $x+4$ is a factor

−4	4	18	23	60
		−16	−8	−60
	4	2	15	0

$4x^2+2x+15$

If $x=-4$

$2(-4)+1 = -7$

$2(-4)+3 = -5$

$2(-4)+5 = -3$

The 3 cons. odd #s are $-7, -5, -3$

Example 4: The product of four integers is $x^4 + 6x^3 + 11x^2 + 6x$ where x is one of the integers. What are possible expressions for the other three integers?

$P(x) = x^4+6x^3+11x^2+6x$

$x(x^3+6x^2+11x+6)$

$P(-1) = (-1)^3+6(-1)^2+11(-1)^1+6$

$= -1+6-11+6$

$= 0$ $(x+1)$ is a factor

−1	1	6	11	6
		−1	−5	−6
	1	5	6	0

x^2+5x+6

$(x+3)(x+2)$

The possible expressions for 3 integers are

$(x+1), (x+3), (x+2)$