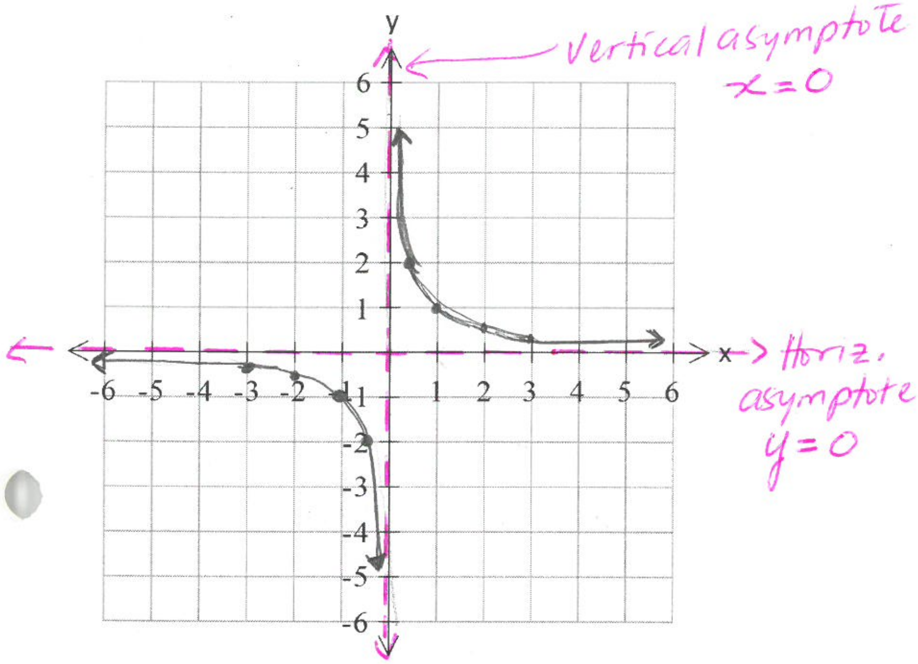
Lesson 4: Identifying Characteristics of Rational Functions

Defn: A rational function has the form $y = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are both polynomial expressions $q(x) \neq 0$.

Non-permissible values are values of x that would make the denominator equal zero.

Example 1: Graph $\boxed{y = \frac{1}{x}}$ using a table of values.



$y = \frac{1}{x}$

x	y
-3	$-1/3 = -0.\overline{3}$
-2	$-1/2 = -0.5$
-1	-1
$-1/2$	-2
0	undefined
$1/2$	2
1	1
2	$1/2$
3	$1/3$

Characteristic	$y = \frac{1}{x}$
Non-permissible value npv	$x = 0$
Equation of vertical asymptote	$x = 0$
Equation of horizontal asymptote	$y = 0$
Domain	$(-\infty, 0) \cup (0, \infty)$
Range	$(-\infty, 0) \cup (0, \infty)$

Notes on asymptotes:

① Vertical Asymptote and Hole (point of discontinuity)

Scenario A: the numerator $p(x)$ and denominator $g(x)$ have no common factors. There will be a vertical asymptote where $g(x) = 0$

Scenario B: the numerator $p(x)$ and denominator $g(x)$ have common factor(s). There will be a "hole."

② Horizontal and Oblique asymptote

(*) Numerator and denominator do not have common factors.

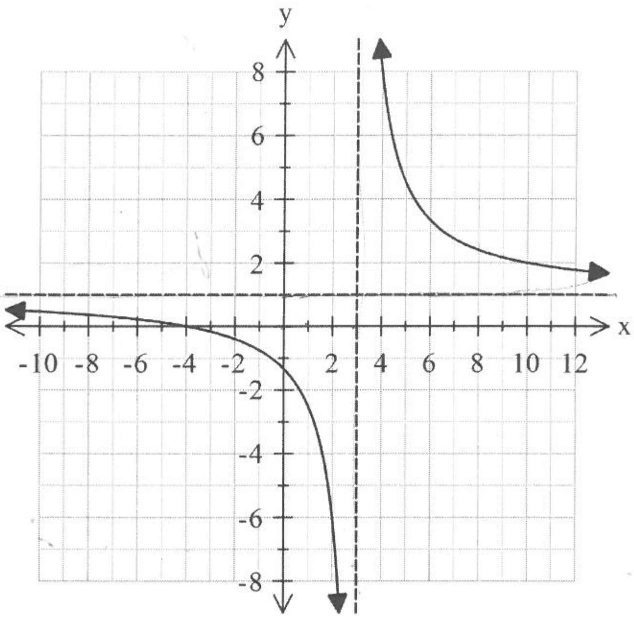
Scenario A: $\deg p(x) < \deg g(x)$ then $\boxed{y=0}$ is the horizontal asymptote.

Scenario B: $\deg p(x) = \deg g(x)$ then $\boxed{y=\frac{a}{b}}$ is the horizontal asymptote, where "a" is the lead coeff of $p(x)$ and "b" is the lead coeff. of $g(x)$.

Scenario C: $\deg p(x) > \deg g(x)$ by 1, then there is an oblique asymptote. To determine the asymptote we have to perform polynomial division.

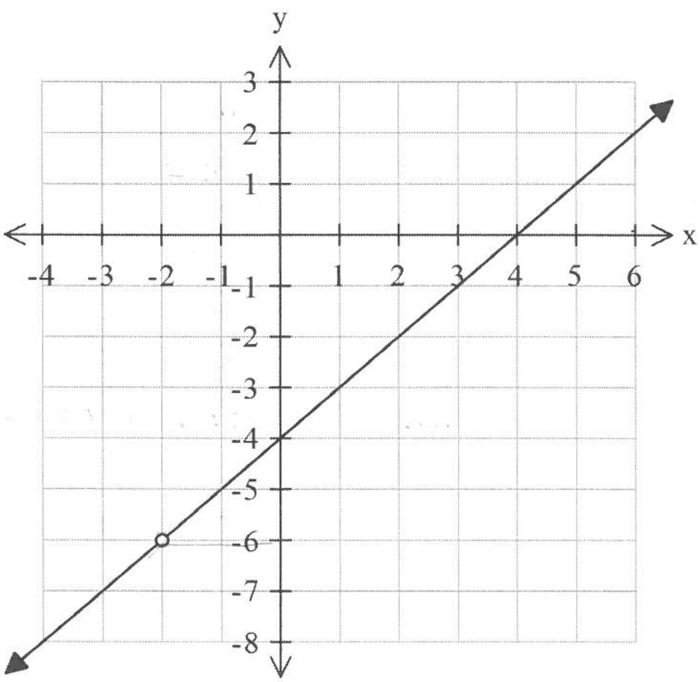
Example 2: State the characteristics of the following rational functions.

a) $y = \frac{x+4}{x-3}$ ← npv



Characteristic	$y = \frac{x+4}{x-3}$
Non-permissible value	$x-3=0$ $x=3$
Equation of vertical asymptote	$x=3$
Equation of horizontal asymptote	$y=1$
Domain	$(-\infty, 3) \cup (3, \infty)$
Range	$(-\infty, 1) \cup (1, \infty)$
Coordinates of hole(s)	No hole.

b) $y = \frac{(x-4)\cancel{(x+2)}}{\cancel{x+2}}$ ← npv

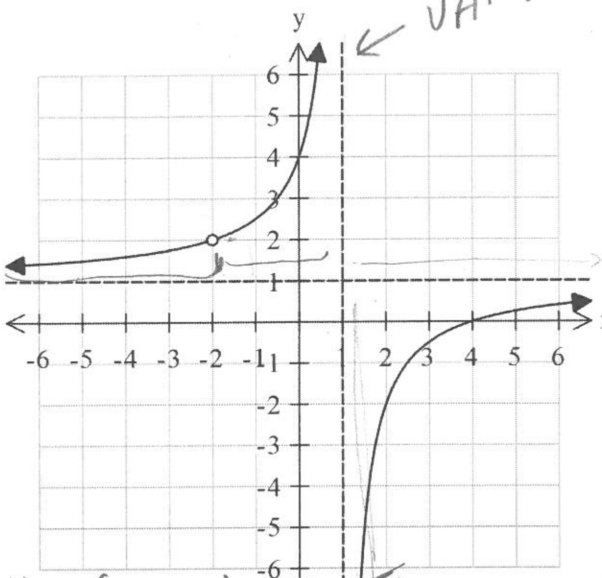


Characteristic	$y = \frac{(x-4)(x+2)}{x+2}$
Non-permissible value	$x=-2$
Equation of vertical asymptote	n/a
Equation of horizontal asymptote	n/a
Domain	$(-\infty, -2) \cup (-2, \infty)$
Range	$(-\infty, -6) \cup (-6, \infty)$
Coordinates of hole(s)	$(-2, -6)$ (x, y)

$\deg P(x) = \deg q(x)$

c) $y = \frac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$

VA: $x=1$



Characteristic	$y = \frac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$
Non-permissible values	$x=-2$ (HOLE) $x=1$ V.A
Equation of vertical asymptote	$x=1$
Equation of horizontal asymptote	$y=\frac{1}{1} \Rightarrow y=1$
Domain $\{x \in \mathbb{R}, x \neq -2, 1\}$	$(-\infty,-2)\cup(-2,1)\cup(1,\infty)$
Range $\{y \in \mathbb{R},$	$(-\infty,1)\cup(1,2)\cup(2,\infty)$ $y \neq 1, 2\}$
Coordinates of hole(s)	$(-2,+2)$

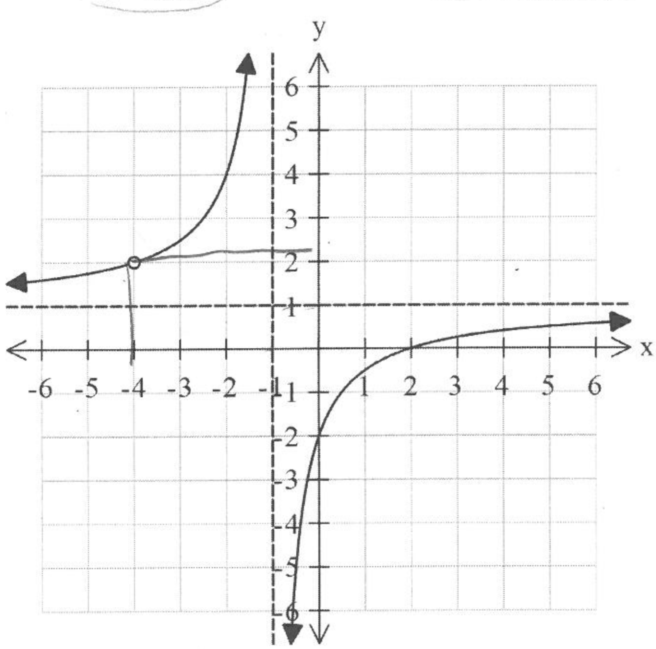
$y = \frac{(x-4)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)}$

$x=-2$

$y = \frac{x-4}{x-1}$

$y = \frac{-2-4}{-2-1} \Rightarrow y = \frac{-6}{-3} = +2$

d) $y = \frac{x^2+2x-8}{x^2+5x+4}$



Characteristic	$y = \frac{x^2+2x-8}{x^2+5x+4}$
Non-permissible values	$x=-1$, VA $x=-4$ (HOLE)
Equation of vertical asymptote	$x=-1$
Equation of horizontal asymptote	$y=1$
Domain $(-\infty,-4)$ $\{x \in \mathbb{R},$	$\cup(-4,-1)\cup(-1,\infty)$ $x \neq -4, -1\}$
Range $(-\infty,1)\cup$ $\{y \in \mathbb{R},$	$(1,2)\cup(2,\infty)$ $y \neq 1, 2\}$
Coordinates of hole(s)	$(-4, 2)$

$y = \frac{x^2+2x-8}{x^2+5x+4}$

$y = \frac{\cancel{(x+4)}(x-2)}{(x+1)\cancel{(x+4)}}$

$\deg p(x) = \deg q(x)$

$y = \frac{a}{b}$

$y = \frac{1}{1}$

$y = 1$

hole at $x=-4$

$y = \frac{x-2}{x+1}$

$y = \frac{-4-2}{-4+1}$

$y = -6/-3 = 2$