

## Pre-Calculus 12: Solving Rational Equations

Solve the following rational equation algebraically. State any non-permissible values for  $x$ .

Example 1:  $\frac{3}{x} = \frac{x-7}{6}$  · NPV:  $x = 0$

$$18 = x^2 - 7x$$

$$0 = x^2 - 7x - 18$$

$$0 = (x - 9)(x + 2)$$

$x = 9$      $x = -2$

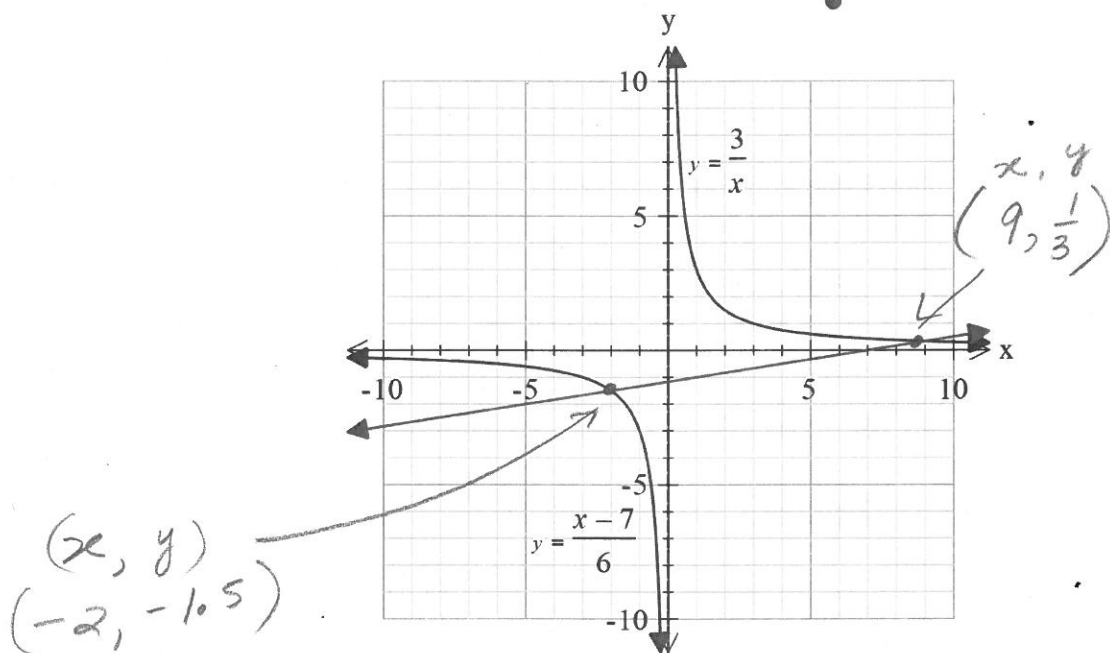
Verify

| LS            | RS              |
|---------------|-----------------|
| $\frac{3}{9}$ | $\frac{9-7}{6}$ |
| $\frac{1}{3}$ | $\frac{2}{6}$   |
|               | $\frac{1}{3}$   |

| LS             | RS               |
|----------------|------------------|
| $\frac{3}{-2}$ | $\frac{-2-7}{6}$ |
|                | $= -\frac{9}{6}$ |
|                | $= -\frac{3}{2}$ |

$x = -2$  ✓

The solution(s) can also be determined graphically. Identify on the graph shown below where the solutions from Example 1 are. Explain how the solutions from Example 1 correspond with the points you identified on the graph.



Example 2: Solve algebraically:  $\frac{3}{2x} - \frac{2x}{x+1} = -2$  State any non-permissible values. NPV:  $x = -1$   $x = 0$

LCD:  $(2x)(x+1)$   
\* multiply each side of equation by LCD, to eliminate the denominator

$$3(x+1) - 2x(2x) = -2(2x)(x+1)$$

$$3x + 3 - 4x^2 = -4x^2 + 4x$$

$$3x + 3 = -4x$$

$$3x + 4x = -3$$

$$7x = -3$$

$$x = -\frac{3}{7}$$

Example 3: Solve algebraically:  $\frac{x}{x-1} - \frac{2x}{1} = \frac{x+1}{2x-2}$ . State any non-permissible values.

$$\frac{x}{x-1} - \frac{2x}{1} = \frac{x+1}{2(x-1)}$$

NPV:  $x = 1$

LCD:  $2(x-1)$

$$2x - 2x(2)(x-1) = x+1$$

$$2x - 4x^2 + 4x = x+1$$

$$0 = 4x^2 - 5x + 1$$

$$0 = (x-1)(4x-1)$$

$\checkmark$   
 $x = 1$   
 extraneous soln

$x = \frac{1}{4}$

Example 4: Solve algebraically:  $\frac{x-1}{x} = \frac{1}{x-1} - \frac{1}{x^2-x}$ . State any non-permissible values.

$$\frac{x-1}{x} = \frac{1}{x-1} - \frac{1}{x(x-1)}$$

NPV:  $x = 0$   $x = 1$

LCD:  $x(x-1)$

$$(x-1)(x-1) = x-1$$

$$x^2 - 2x + 1 = x - 1$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$x = 2$

$x = 1$   
 extraneous solution