Example 3: Use the equations of the following rational functions to determine non-permissible values, equations of any vertical and horizontal asymptotes, and coordinates of any hole.

a) $y = \frac{x^2-25}{x+5}$

$y = \frac{\cancel{(x+5)}(x-5)}{\cancel{x+5}}$

NPV: $x = -5$ (Hole)

VA: No VA

HA: none

just an oblique line

coordinate of hole

if $x = -5$

$y = x - 5$

$y = -5 - 5$

$y = -10$ ∴ "hole" at $(-5, -10)$

b) $y = \frac{5x+3}{x-2}$

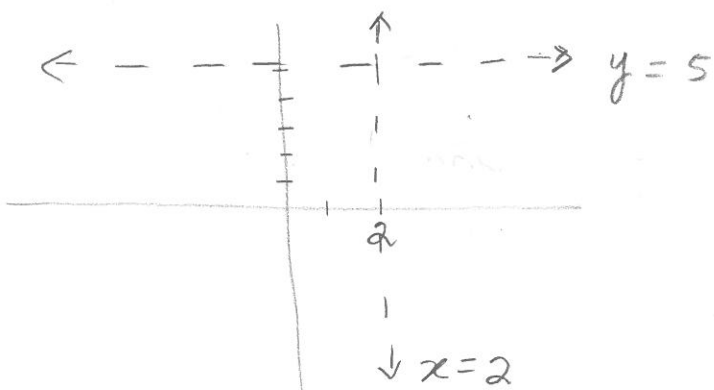
NPV: $x = 2$

VA: $x = 2$

No hole

HA: $y = \frac{5}{1}$

$y = 5$



c) $y = \frac{x^2-x-2}{x+1}$

$y = \frac{(x-2)\cancel{(x+1)}}{\cancel{(x+1)}}$

NPV: $x = -1$

NO V.A

Hole at $x = -1$

$y = x - 2$

$y = -1 - 2$

$y = -3$

∴ hole at $(-1, -3)$

just an oblique line

$y = x - 2$

d) $y = \frac{x^2+6x-7}{x+2}$

$$y = \frac{(x-1)(x+7)}{x+2}$$

NPV: $x = -2$

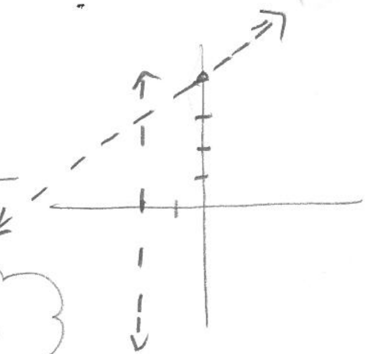
VA: $x = -2$

No hole

deg $p(x) >$ deg $q(x)$

$$\begin{array}{r|rrr} -2 & 1 & 6 & -7 \\ & & -2 & -8 \\ \hline & 1 & 4 & -15 \end{array}$$

$x + 4 \quad R \ \frac{-15}{x+2}$



Oblique asymptote at $y = x + 4$

e) $y = \frac{x+2}{x^2+6x-7}$

$$y = \frac{x+2}{(x+7)(x-1)}$$

NPV: $x = -7$ and $x = 1$

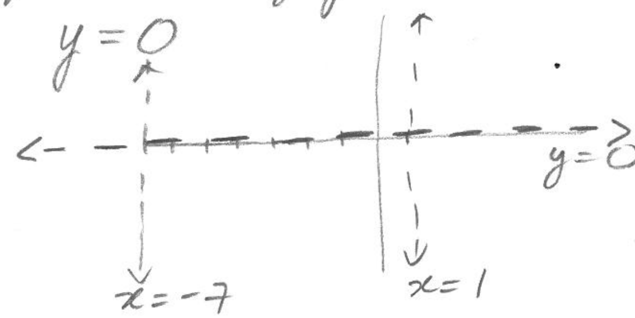
No hole

VA: $x = -7$ and $x = 1$

HA: $y = 0$

deg $p(x) <$ deg $q(x)$

$y = 0$



f) $y = \frac{6}{x+3}$

NPV: $x = -3$

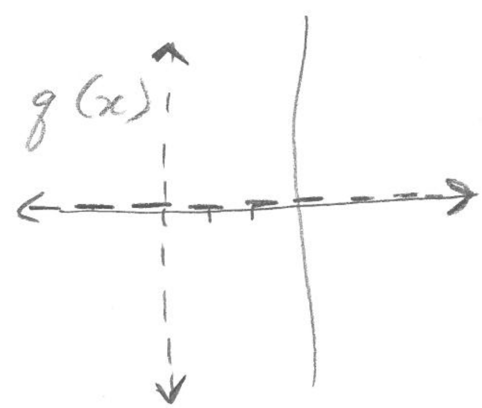
VA: $x = -3$

No hole

HA: $y = 0$

deg $p(x) <$ deg $q(x)$

$y = 0$



Assignment Time! Work on p.104- 1 – 3

p.114- 4a)andb), 5, 6, 9 (omit graphs A and F), 10 i) and ii), MC 2

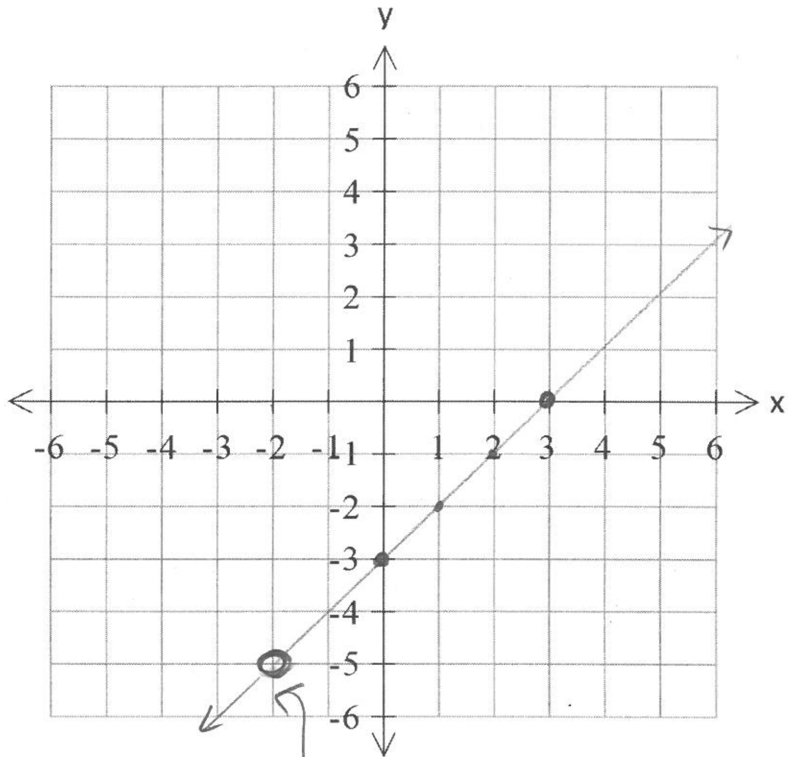
Lesson 5: Sketching the Graph of a Rational Function

To sketch the graph of a rational function, determine and state:

- The non-permissible values of x (holes and vertical asymptotes).
- The equation of the horizontal asymptote (if there is one.)
- The intercepts.

Example 1: Sketch the graph of $y = \frac{x^2-x-6}{x+2}$

$y = \frac{(x-3)\cancel{(x+2)}}{\cancel{(x+2)}}$



hole at $(-2, -5)$

NPV: $x = -2$

hole at $x = -2$

$y = x - 3$

$y = -2 - 3$

$y = -5$

$(-2, -5)$

HA: $\deg p(x) > \deg q(x)$

if we divide we will end up w/ exactly

$y = x - 3$ no remainder

To calculate the y-int, set $x = 0$

$y = \frac{0^2 - 0 - 6}{0 + 2}$

$y = \frac{-6}{+2}$

$y = -3$

To calculate x-int

set $y = 0$

$0 = \frac{x^2 - x - 6}{x+2}$

$0 = x^2 - x - 6$

$0 = (x+2)(x-3)$

$x = -2$ npv $\quad \boxed{x = +3}$