## Lesson 3: Introduction to Composite Functions

Example 1: The tables below define two functions. Use these tables to determine the values requested below the tables.

Work from inside to outside

| $x$ | $f(x)$ |
|---|---|
| $-2$ | 8 |
| $-1$ | 3 |
| 0 | 0 |
| 1 | $-1$ |
| 2 | 0 |

| $x$ | $g(x)$ |
|---|---|
| $-2$ | 3 |
| $-1$ | 2 |
| 0 | 1 |
| 1 | 0 |
| 2 | 1 |

a) $f(g(2))$

$= f(1)$

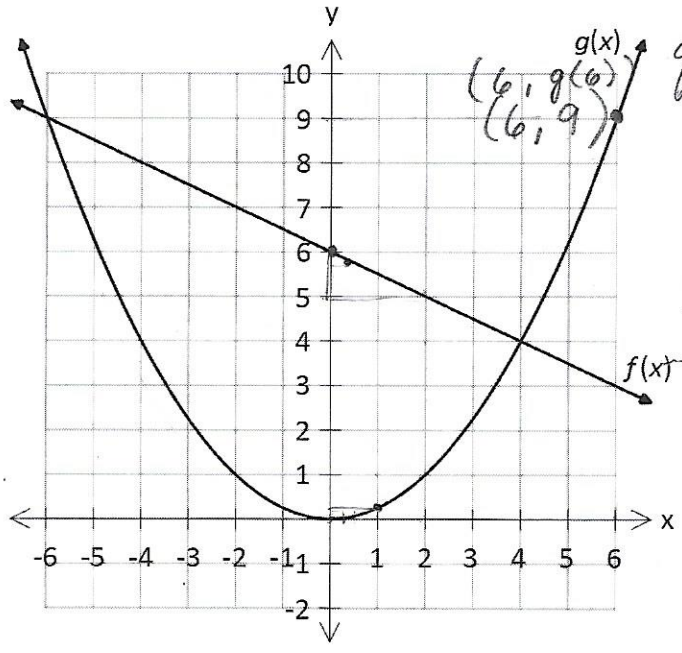
$= -1$

b) $g(g(-1))$

$= g(2)$

$= 1$

Example 2: Given the graphs of $y = f(x)$ and $y = g(x)$, determine the values requested below the graphs.




quadratic function

$y = \frac{1}{4}x^2$

$y = ax^2$ pick a point on parabola (2, 1)

$1 = a(2)^2$

$\frac{1}{4} = \frac{4a}{4} \Rightarrow a = \frac{1}{4}$

Linear function

y int = 6

slope $= \frac{-1}{2}$

$y = -\frac{1}{2}x + 6$

a) $g(f(0))$

$= g(6)$

$= 9$

b) $f(g(1))$ ←

$= f\left(\frac{1}{4}(1)^2\right)$

$= f\left(\frac{1}{4}\right)$

$= -\frac{1}{2}\left(\frac{1}{4}\right) + 6 = -\frac{1}{8} + 6 = 5\frac{7}{8}$

$g(x) = \frac{1}{4}x^2$

$f(x) = -\frac{1}{2}x + 6$

Example 3: Given the functions $h(x) = \sqrt{x+5}$ and $m(x) = (x-1)^2$, determine the values requested below:

a) $m(h(4))$

$= m(\sqrt{4+5})$

$= m(\sqrt{9})$

$= m(3)$

$= (3-1)^2 = 2^2 = \boxed{4}$

b) $h(m(13))$

$= h((13-1)^2)$

$= h(12)^2$

$= h(144)$

$= \sqrt{144+5}$

$= \boxed{\sqrt{149}}$

Example 4: Given $f(x) = x^2 + 3x$ and $g(x) = 3x - 5$, determine an explicit equation for each requested composite function, and state the domain of each composite function. (**OPTIONAL** Use graphing technology to graph each composite function and determine the range.) * See other page for Full solution.

a) $f(g(x))$

$9x^2 - 21x + 10x$

D: $x \in \mathbb{R}$ or $(-\infty, \infty)$

b) $g(f(x))$

$3x^2 + 9x - 5$

D: $x \in \mathbb{R}$ or $(-\infty, \infty)$

c) $f(f(x))$

$x^4 + 6x^3 + 12x^2 + 9x$

D: $x \in \mathbb{R}$ or $(-\infty, \infty)$

* May use Desmos as a graphing tool.

**Assignment Time!** Work on p.298- 4 – 11, MC 1&2

pg 10 Example #4

$f(x) = x^2 + 3x$ and $g(x) = 3x - 5$

a) $f(g(x))$

$= f(3x-5)$

Note: replace "x" from f(x) w/ $3x-5$

$= (3x-5)^2 + 3(3x-5)$

$= 9x^2 - 30x + 25 + 9x - 15$

$= \boxed{9x^2 - 21x + 10x}$

b) $g(f(x))$

$= g(x^2+3x)$

Note: replace "x" from g(x) w/ $x^2+3x$

$= 3(x^2+3x) - 5$

$= \boxed{3x^2 + 9x - 5}$

c) $f(f(x)) = f(x^2+3x)$

Note: replace "x" in f(x) w/ $x^2+3x$

$= (x^2+3x)^2 + 3(x^2+3x)$

$= x^4 + 6x^3 + 9x^2 + 3x^2 + 9x$

$= \boxed{x^4 + 6x^3 + 12x^2 + 9x.}$