Lesson 4 - Composite functions

Ex 1) $f(x) = 2x - 1$ $g(x) = x^2 - 2$

a) $D: x \in \mathbb{R}$ or $(-\infty, \infty)$
Domain of $f(x)$

Domain of $g(x)$
$D: x \in \mathbb{R}$ or $(-\infty, \infty)$

b) $y = g(f(x))$

$y = g(2x-1)$

Note: replace "x" in $g(x)$ with $(2x-1)$

$= (2x-1)^2 - 2$

$= 4x^2 - 4x + 1 - 2$

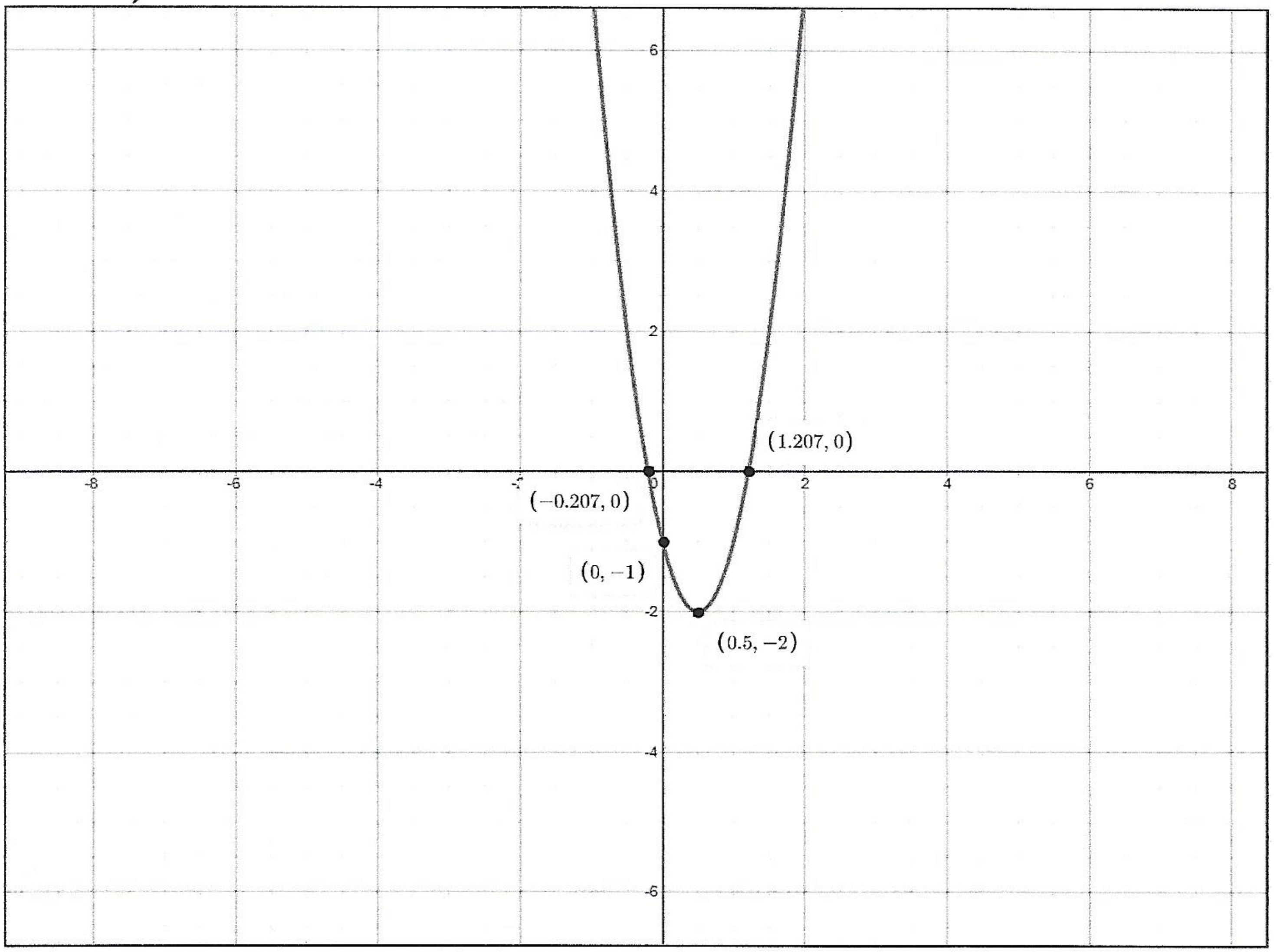
$y = 4x^2 - 4x - 2$

The composite is a quadratic function

Therefore the domain is

$\boxed{x \in \mathbb{R} \text{ or } (-\infty, \infty)}$

Ex 1b)



$y = 4x^2 - 4x - 1$

Ex 1 c)

$$g(x) = x^2 - 2$$

$y = g(g(x))$

$y = g(x^2 - 2)$

Note: Replace "x" in $g(x)$ w/ $(x^2 - 2)$

$y = (x^2 - 2)^2 - 2$

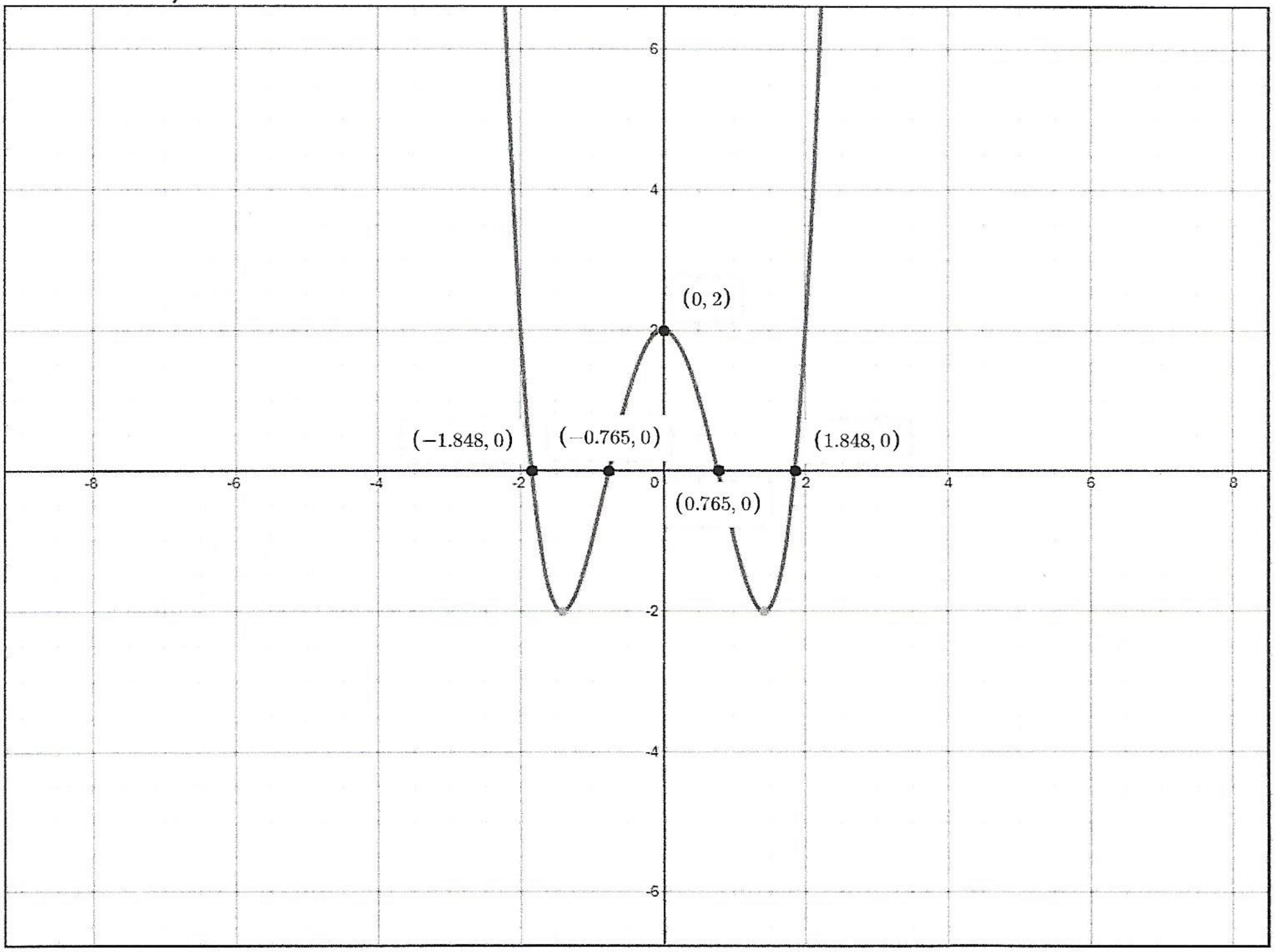
$y = x^4 - 4x^2 + 4 - 2$

$y = x^4 - 4x^2 + 2$

The composite function is a quartic function.

D: $x \in \mathbb{R}$ or $(-\infty, \infty)$

Ex 1c)



$y = x^4 - 4x^2 + 2$

Ex 2) a)

$$h(x) = \frac{1}{x-2} \qquad j(x) = x^2 - x$$

$$j(h(x))$$

$$= j\left(\frac{1}{x-2}\right)$$

Note: Replace "x" in $j(x)$ w/ $\frac{1}{x-2}$

$$= \left(\frac{1}{x-2}\right)^2 - \left(\frac{1}{x-2}\right)$$

$$= \frac{1}{(x-2)^2} - \frac{1}{x-2}$$

$$= \frac{1}{(x-2)^2} - \frac{1\,(x-2)}{(x-2)(x-2)}$$

$$= \frac{1}{(x-2)^2} - \frac{(x-2)}{(x-2)^2}$$

$$= \frac{1 - (x-2)}{(x-2)^2}$$

$$= \frac{1 - x + 2}{(x-2)^2}$$

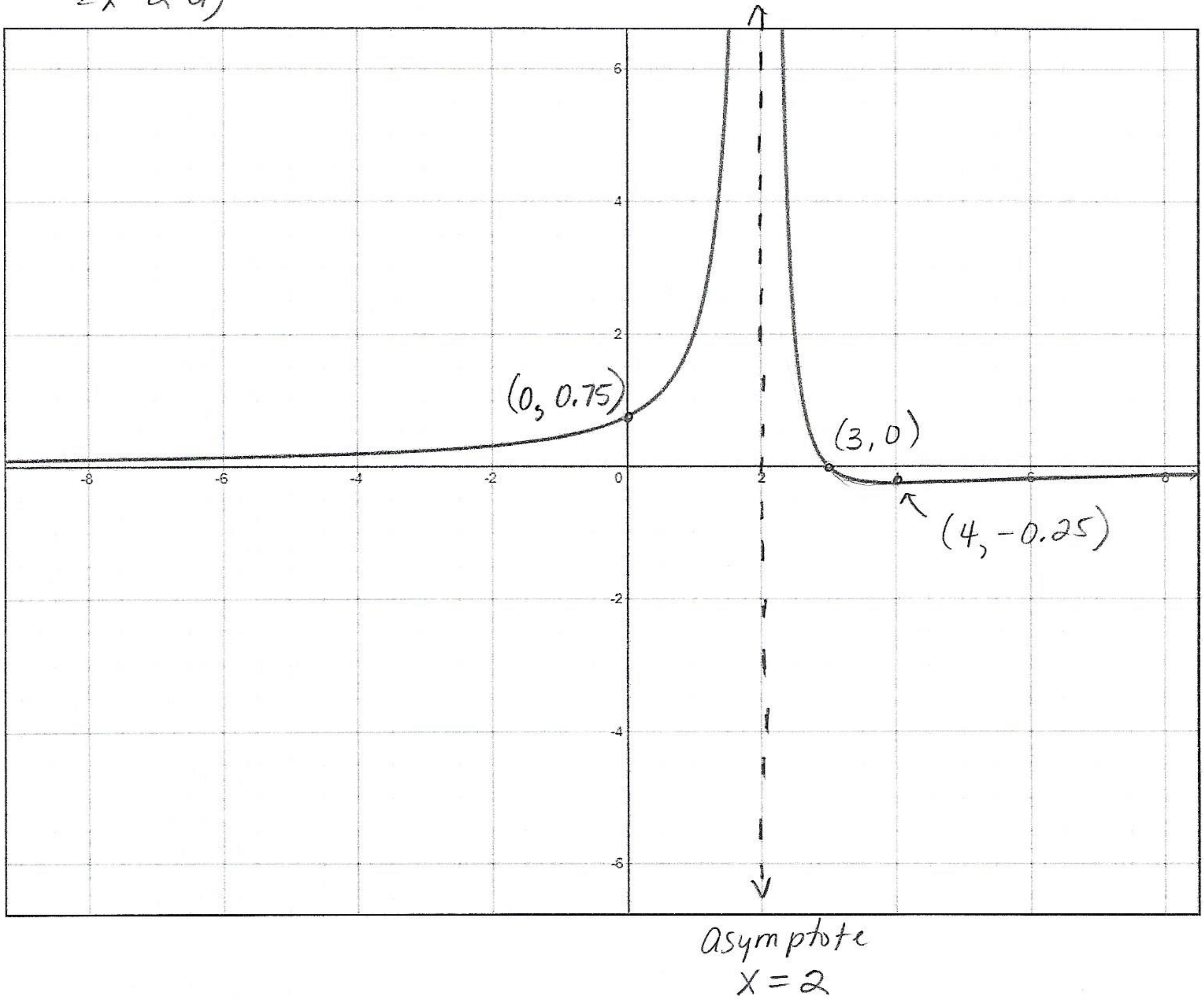
$$y = \frac{-x+3}{(x-2)^2}$$

Since this gives us a rational function, there is a npv that will make denominator zero

npv: $x = 2$

Domain: $x \in \mathbb{R},\ x \neq 2$

Ex 2 a)



$y = \frac{1}{(x-2)^2} - \frac{1}{(x-2)}$

Ex 2) b) $h(x) = \frac{1}{x-2}$ $\quad j(x) = x^2 - x$

$y = h(j(x))$

$y = h(x^2 - x)$

Note: replace "x" in $h(x)$ w/ $x^2 - x$

$y = \frac{1}{(x^2 - x) - 2}$

$y = \frac{1}{x^2 - x - 2}$

The composite gives us a rational function

$y = \frac{1}{(x-2)(x+1)}$

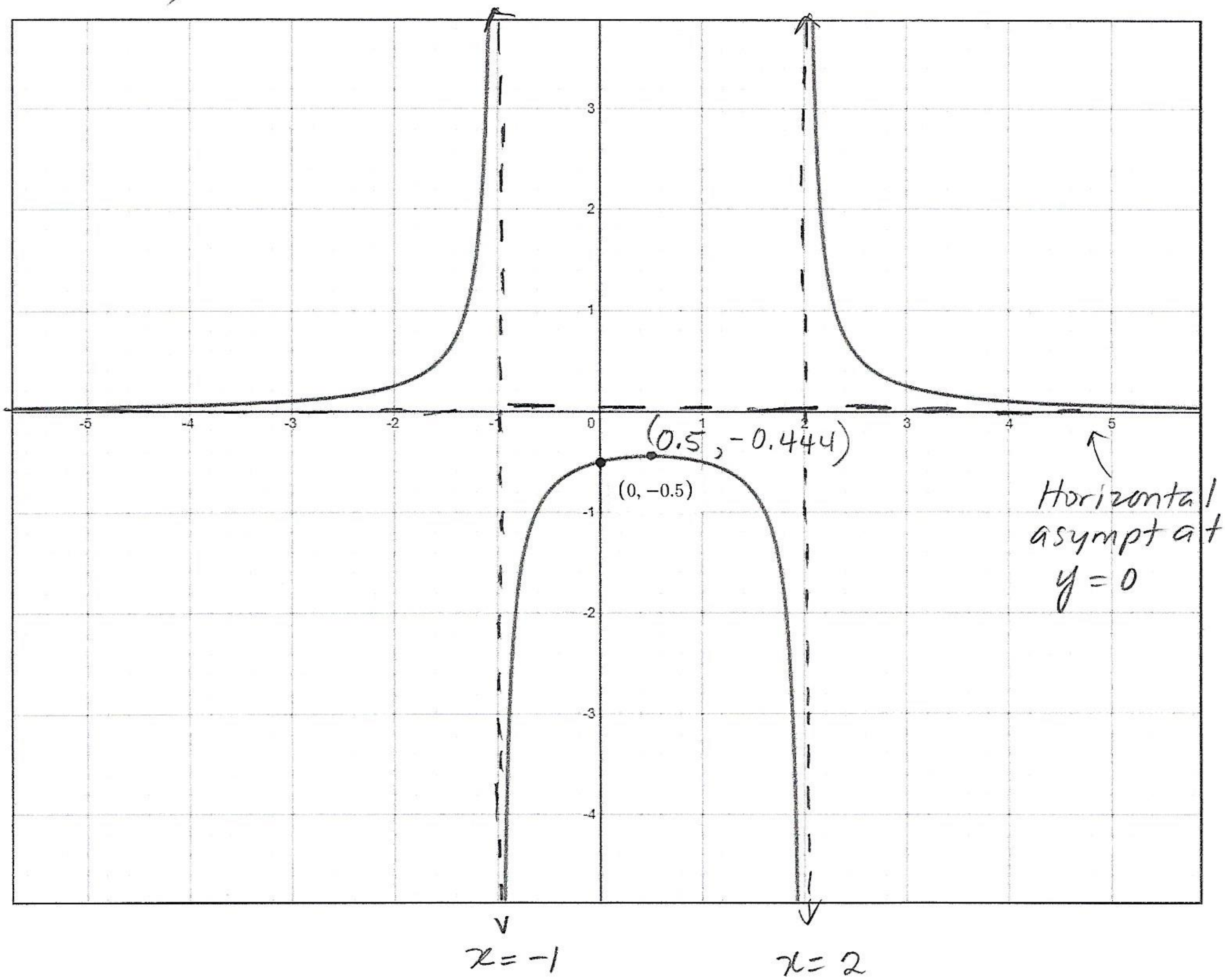
There are npv.

$x = 2$ and $x = -1$

Therefore the Domain is $x \in \mathbb{R}$, $x \neq 2$ and $x \neq -1$

There are vertical asymptotes at $x = 2$ and $x = -1$

Ex 2b)



$y = \frac{1}{(x-2)(x+1)}$

Ex 3) a) $f(x) = \sqrt{x}$ $\quad g(x) = -x^2 + 2x$.

$y = g(f(x))$

$y = g(\sqrt{x})$

Note: Replace "x" in g(x) w/ $\sqrt{x}$

$y = -(\sqrt{x})^2 + 2\sqrt{x}$

$y = -x + 2\sqrt{x}$

What are the possible x-values?

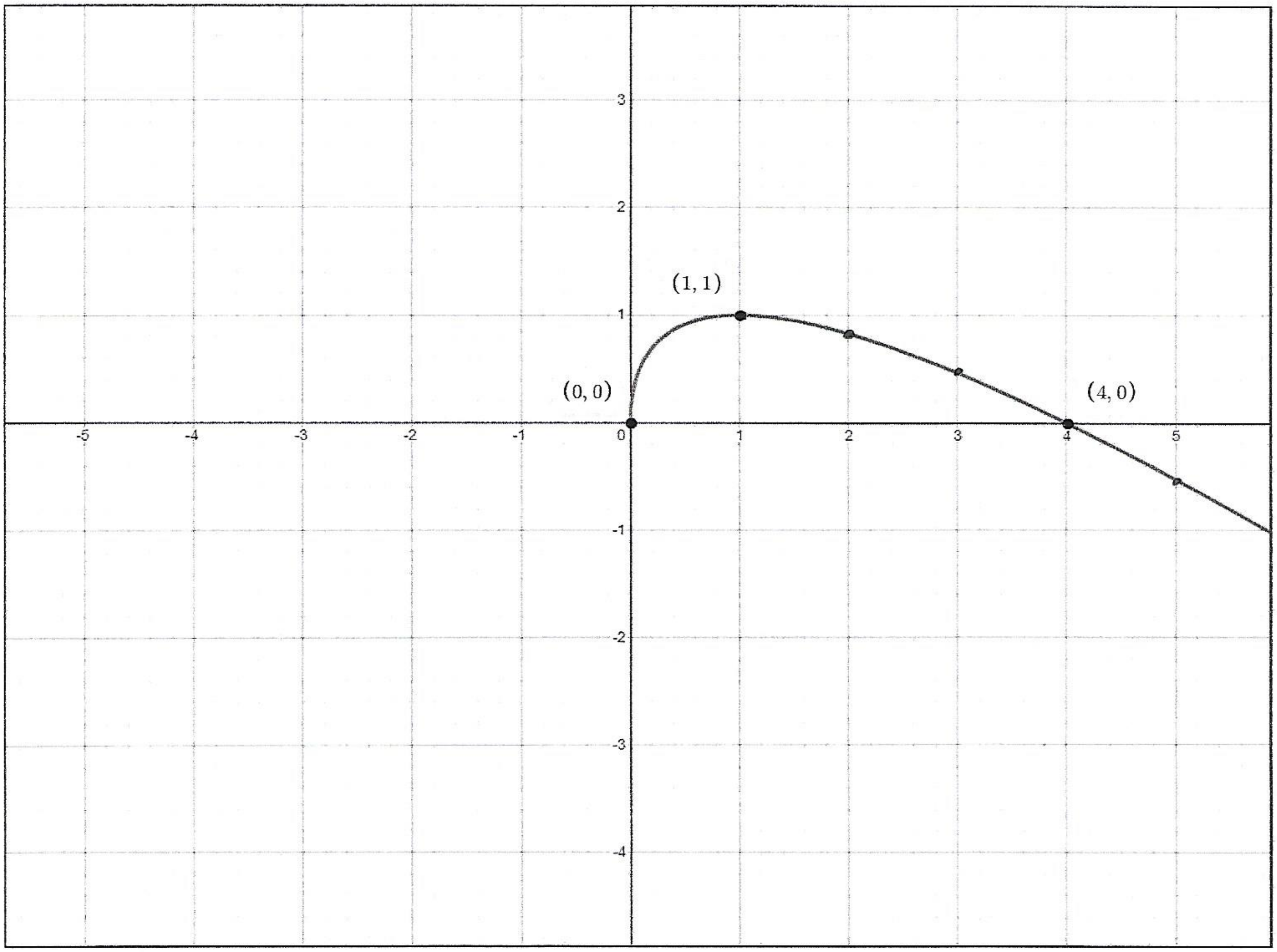
$x \geq 0$.

The domain is $x \geq 0$.

x	$-x$	$2\sqrt{x}$	$-x + 2\sqrt{x}$
0	0	0	0
1	-1	2	1
2	-2	~ 2.8	0.8
3	-3	~ 3.5	0.5
4	-4	4	0
5	-5	4.5	-0.5

← Combining Function learned in Lesson 1

Ex 3 a)



$y = -x + 2\sqrt{x}$

Ex 3 b)

$f(x) = \sqrt{x}$ $\qquad$ $g(x) = -x^2 + 2x$

$y = f(g(x))$

$y = f(-x^2 + 2x)$

Note: Replace "x" in $f(x)$ w/ $(-x^2 + 2x)$

$y = \sqrt{-x^2 + 2x}$

$y = \sqrt{-x(x-2)}$

The composite function is a radical function.

$-x^2 + 2x \geq 0$

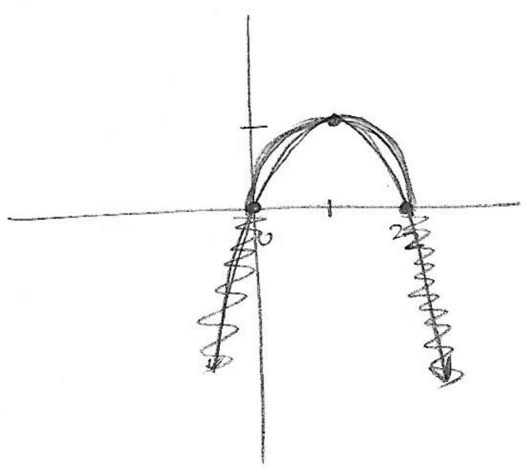
$-x(x-2) \geq 0$

We could use the strategy we learned in CH 2.

- First graph Quad Function

$-x^2 + 2x$

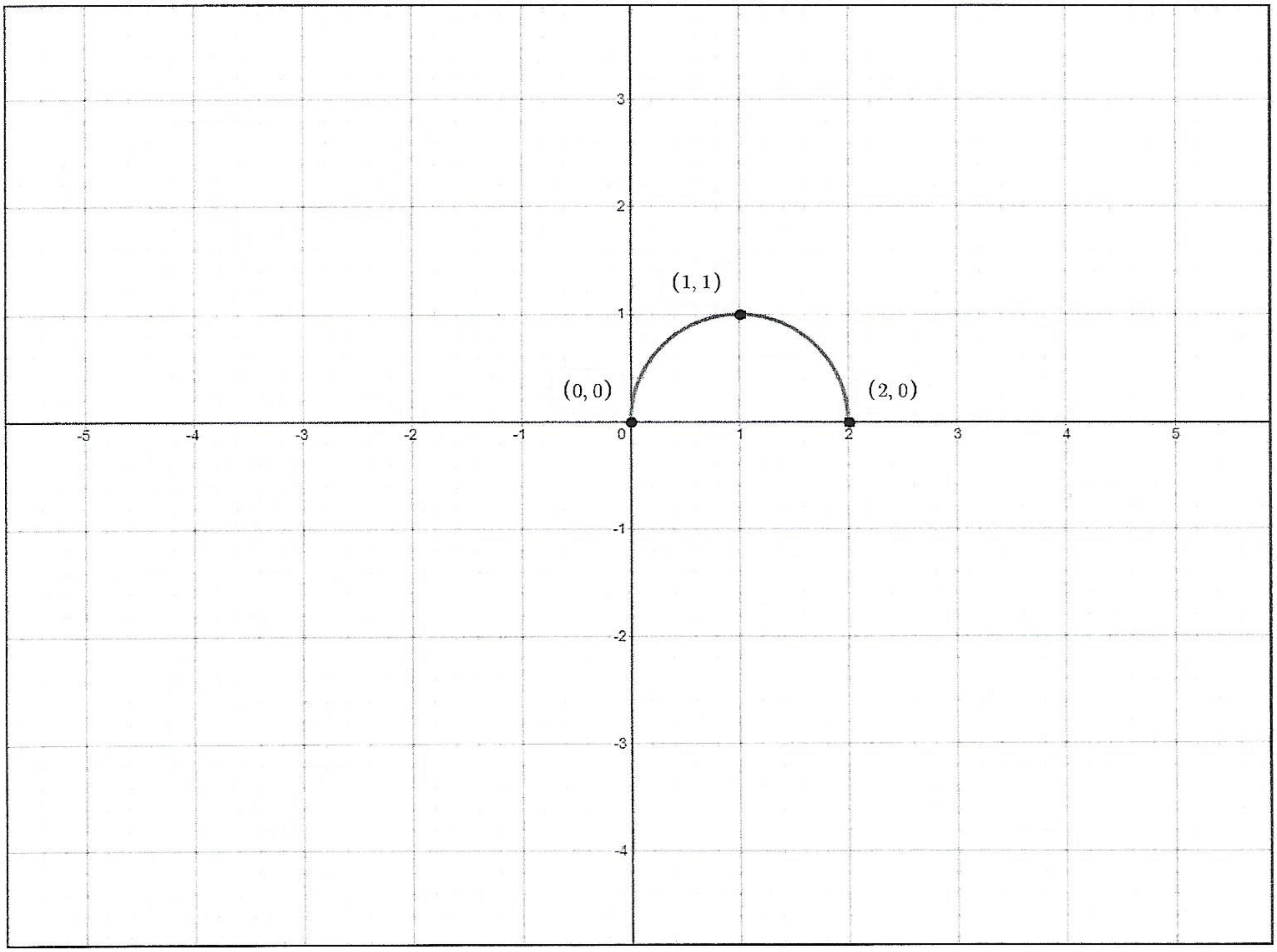
$-x(x-2) = 0$



- We can apply the squareroot

Domain: $[0, 2]$

Ex 3b)



$y = \sqrt{-x^2 + 2x}$

Example 3: Given the functions $f(x) = \sqrt{x}$ and $g(x) = -x^2 + 2x$, determine an explicit equation for each composite function below, then state its domain.

a) $g(f(x))$

b) $f(g(x))$

OPTIONAL Verify your answers using graphing technology.

Example 4: For each function, determine possible functions f and g so that $y = f(g(x))$.

a) $y = \frac{1}{\sqrt{x}}$

$f(x) = \frac{1}{x}$

$g(x) = \sqrt{x}$

$f(g(x)) = \frac{1}{\sqrt{x}}$

b) $y = |2x - 1|^5$

$g(x) = 2x - 1$

$f(x) = |x|^5$

$y = f(g(x))$

DONE.

Assignment Time! Work on p.314- 3, 4, 6, 8a, 10, MC 1&2