$y = a f(b(x - c)) + d$

Lesson 2: Determining characteristics of the graphs of $y = \sin x$ $y = \cos x$

The general form of a sinusoidal function is $y = a\sin b(x - c) + d$. The parameters of $a, b, c,$ *and* d transform the size and shape of the graph.

Amplitude of a Sinusoidal Function

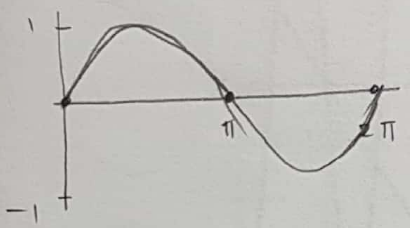
The "a" value in a sinusoidal equation controls the vertical stretch. If the "a" value is anything other then 1, the "a" value will appear as a coefficient of the *sin* or *cos*. If the "a" value is negative, the graph is also reflected in the x –axis.

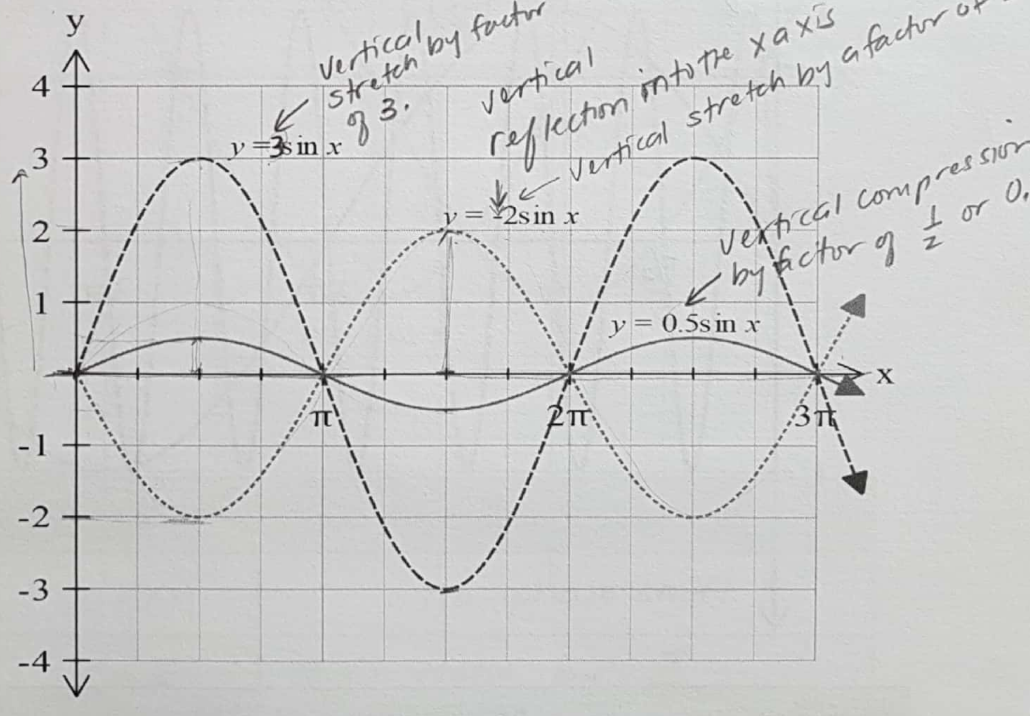
distance from midline to max (or min)

$|a|$ represents the amplitude of a sinusoidal function. If $a < 0$, the function is also reflected in the x-axis. Amplitude can be determined from inspecting the graph or by using the formula:

$$amplitude = \frac{|maximum\ value - minimum\ value|}{2}$$

Study the graphs of $y = 3sin\ x$ and $y = -2\sin x$ and $y = 0.5\sin x$ then and complete the table below.





Function	Amplitude	Domain	Range
$y = 3\sin x$	amp = 3	[0, ∞)	[−3, 3]
$y = -2\sin x$	amp = 2	[0, ∞)	[−2, 2]
$y = 0.5\sin x$	amp = 0.5	[0, ∞)	[−0.5, 0.5]

b is horizontal stretch / compression by a factor of $\frac{1}{b}$

Period of a Sinusoidal Function

$y = a \sin b(x - c) + d$

A function that repeats its values in regular intervals over its domain is a periodic function. The graphs of $y = \sin x$ and $y = \cos x$ are both periodic functions. The period is the horizontal length of one interval or cycle on a periodic graph. From the general equation of a sinusoidal function, $y = a\sin b(x - c) + d$ the parameter b represents the frequency of the cycle within an interval of 2π, therefore, b affects the period of the graph. The period can be determined from inspecting the graph or by using the formula:

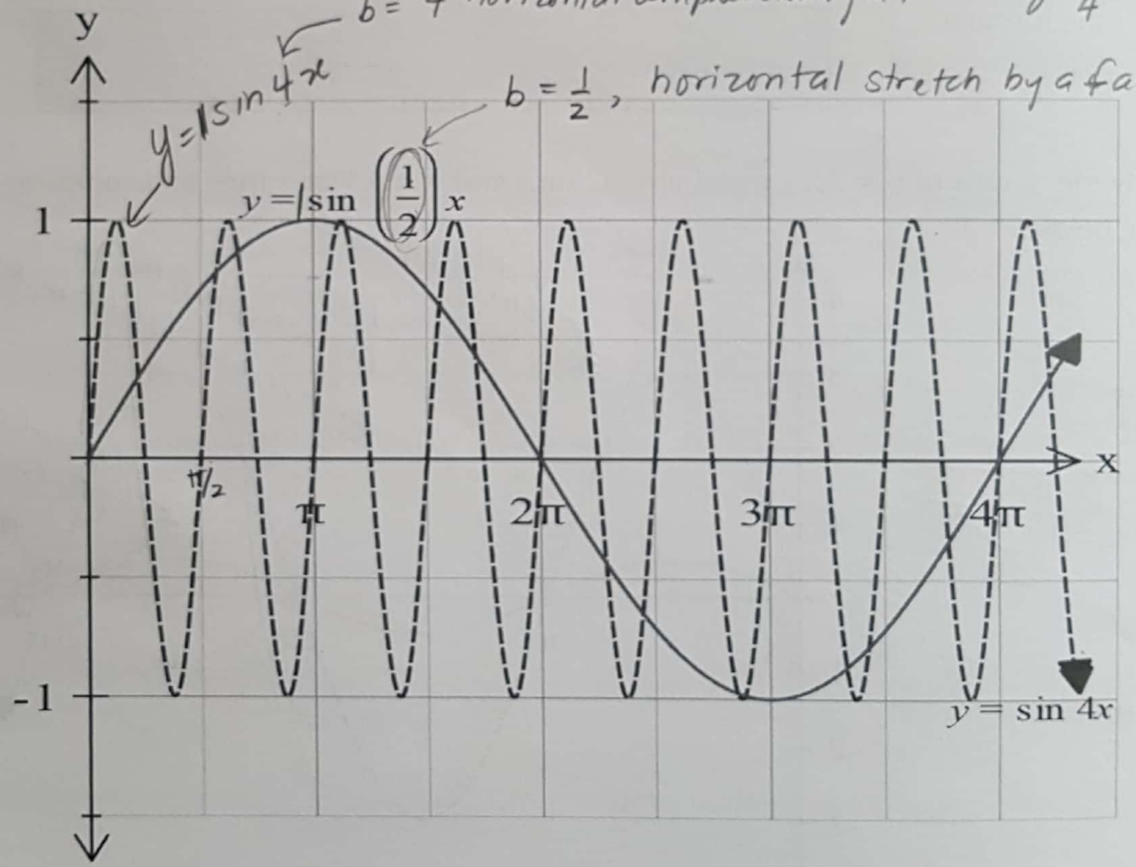
$P = \frac{2\pi}{b}$ $\quad$ $b = \frac{2\pi}{P}$

$$\text{Period} = \frac{2\pi}{b} \quad \text{or} \quad \text{Period} = \frac{360^\circ}{b}$$

Study the graphs of $y = \sin 4x$ and $y = \sin\frac{1}{2}x$ then complete the table below.

$b = 4$ horizontal compression by a factor of $\frac{1}{4}$

$b = \frac{1}{2}$, horizontal stretch by a factor of $\frac{1}{\frac{1}{2}} = 2$

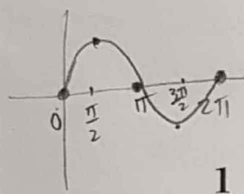


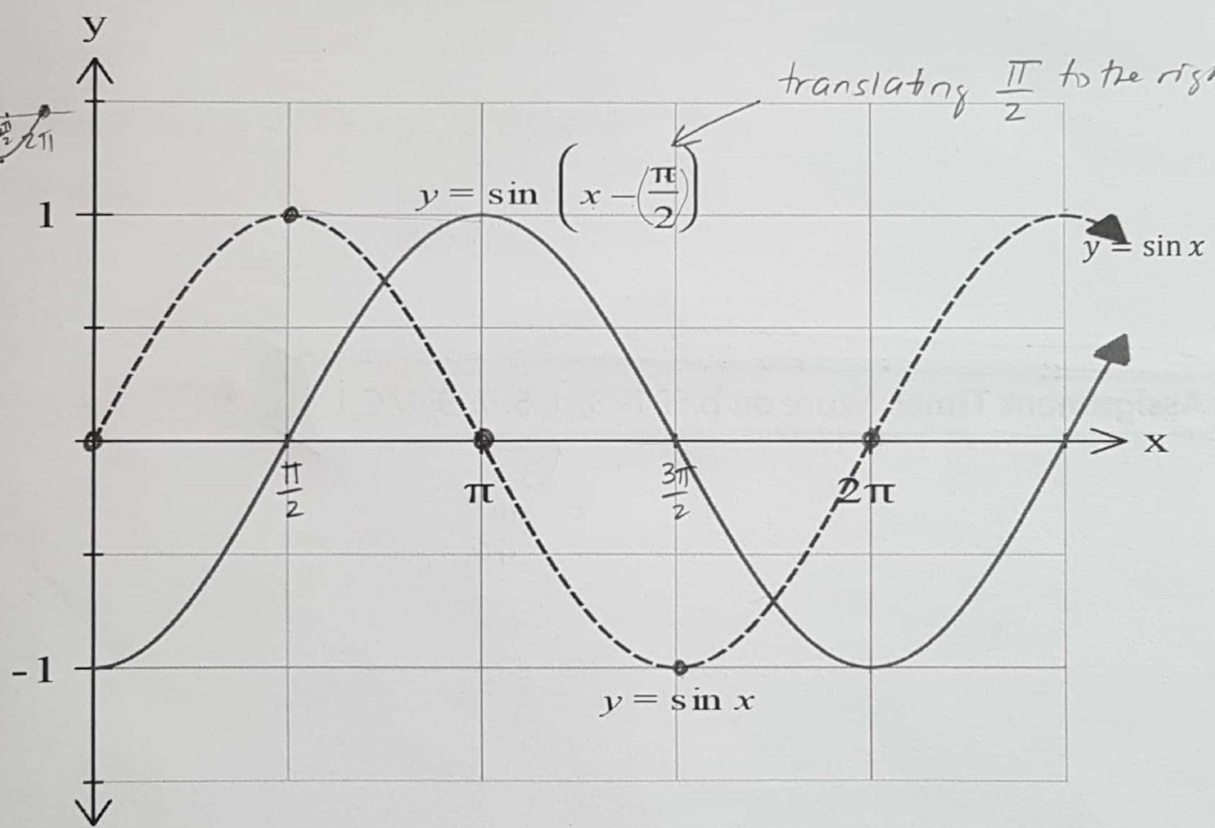
Function	Amplitude	Domain	Range	Period
$y = \sin 4x$	$a = 1$	$[0, \infty)$	$[-1, 1]$	$P = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$
$y = \sin 0.5x$	$a = 1$	$[0, \infty)$	$[-1, 1]$	$P = \frac{2\pi}{b} = \frac{2\pi}{0.5} = 4\pi$

Phase Shift of a Sinusoidal Function

From the general equation of a sinusoidal function, $\boxed{y = a \sin b(x - c) + d}$ the parameter c controls the phase(horizontal) shift. In general, the graph of $y = \sin(x - c)$ is the image after the graph of $y = \sin x$ has been translated c units horizontally.

Study the graphs of $\boxed{y = \sin x}$ and $y = \sin(x - \frac{\pi}{2})$ and $y = \sin(x + \frac{\pi}{4})$ and describe the transformations resulting from the phase shift.





Function	Value of "c"	Description of phase shift
$y = \sin(x - 0)$	$c = 0$	no phase shift
$y = \sin(x - \frac{\pi}{2})$	$c = \frac{\pi}{2}$	shift right by $\frac{\pi}{2}$
$y = \sin(x + \frac{\pi}{4})$	$c = -\frac{\pi}{4}$	shift left by $\frac{\pi}{4}$
$y = \cos(x - \frac{\pi}{6})$	$c = \frac{\pi}{6}$	shift right by $\frac{\pi}{6}$
$y = \cos(x + \pi)$ $y = \cos(x - -\pi)$	$c = -\pi$	shift left by π

Vertical Shift of a Sinusoidal Function

From the general equation of a sinusoidal function, $y = a \sin b(x - c) + d$ the parameter "d" controls the vertical shift. If "d" is positive, the graph is shifted upwards. If "d" is negative, the graph is shifted downward.

Assignment Time! Work on p.521- 3, 4, 5, 7, 9, MC 1

omit
4c