

$$y = a \sin(b(x - c)) + d$$

$$y = a \cos(b(x - c)) + d$$

a = amplitude [vertical stretch or compression, also reflection into the x -axis]

b = this will help us determine the period.

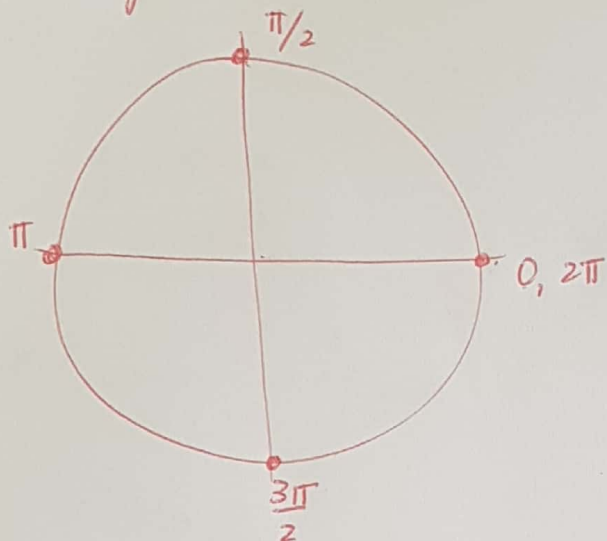
$$P = \frac{2\pi}{b} \quad \left[\begin{array}{l} \text{The "b" tells us the} \\ \text{horizontal stretch/compression,} \\ \text{and also horizontal reflection} \\ \text{in to y-axis} \end{array} \right]$$

c = phase shift [horizontal translation either to left or right]

d = where the middle (median line) is. [vertical translation either up or down]

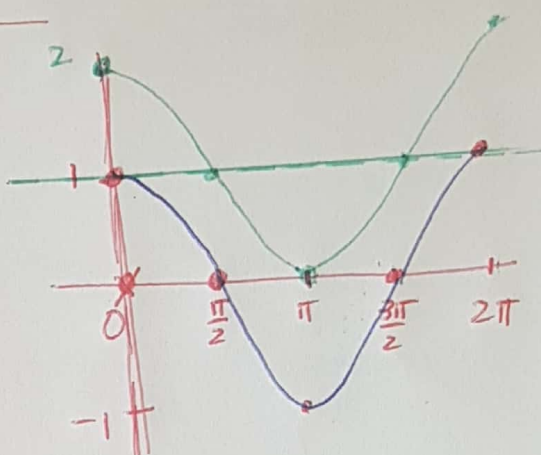
#7a) PAGE 524

$$y = \cos x$$



(x, y)
$(0, 1)$
$(\frac{\pi}{2}, 0)$
$(\pi, -1)$
$(\frac{3\pi}{2}, 0)$
$(2\pi, 1)$

$$y = \cos x + 1$$



Using mapping method.

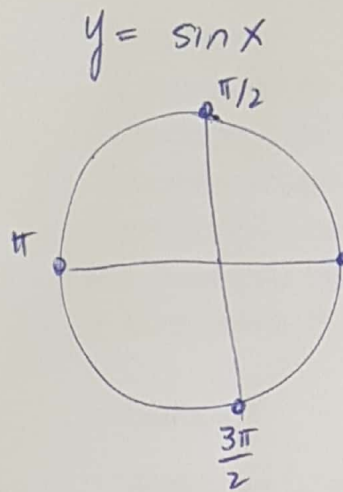
(x, y)	$(x, y+1)$
$(0, 1)$	$(0, 2)$
$(\frac{\pi}{2}, 0)$	$(\frac{\pi}{2}, 1)$
$(\pi, -1)$	$(\pi, 0)$
$(\frac{3\pi}{2}, 0)$	$(\frac{3\pi}{2}, 1)$
$(2\pi, 1)$	$(2\pi, 2)$

$$(x, y) \rightarrow \left(\frac{1}{b}x + c, ay + d \right)$$

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 7b) $y = \sin 2x$

$$b = 2 \Rightarrow p = \frac{2\pi}{2} = \pi$$

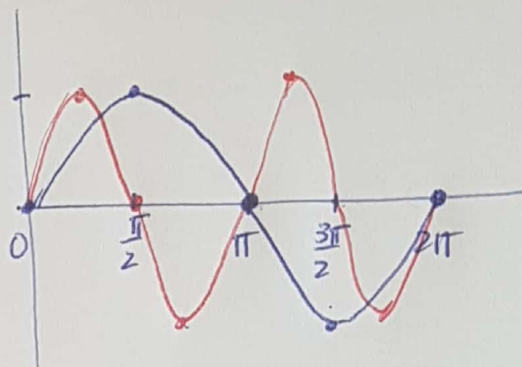
for 1 cycle, it would take π radians.



$$\Rightarrow$$

(x, y)
$(0, 0)$
$(\frac{\pi}{2}, 1)$
$(\pi, 0)$
$(\frac{3\pi}{2}, -1)$
$(2\pi, 0)$

\Rightarrow



If we are using mapping.

(x, y)	$(\frac{x}{2}, y)$
$(0, 0)$	$(0, 0)$
$(\frac{\pi}{2}, 1)$	$(\frac{\pi}{4}, 1)$
$(\pi, 0)$	$(\frac{\pi}{2}, 0)$
$(\frac{3\pi}{2}, -1)$	$(\frac{3\pi}{4}, -1)$
$(2\pi, 0)$	$(\pi, 0)$

$$b = 2$$

$$(x, y) \rightarrow \left(\frac{1}{b}x + c, ay + d\right)$$

Lesson 3: Sketching transformations of the graphs of $y = \sin x$ $y = \cos x$

Recall that:

a : represents the vertical stretch or compression (affects the amplitude). If a is negative, the graph is also reflected over the x -axis.

b : represents the horizontal stretch or compression (affects the period). If b is negative, the graph is also reflected over the y -axis.

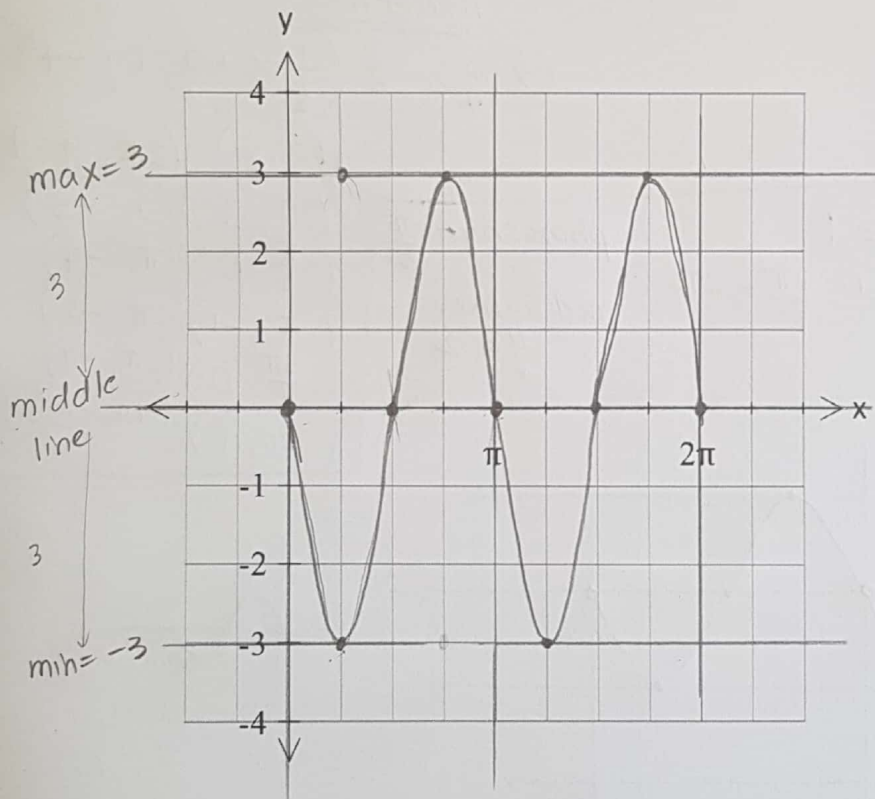
c or h : represents the horizontal shift (affects the "starting point").

d or k : represents the vertical shift (affects the median).

Example 1

Sketch two cycles of the graph of each function.

a) $y = -3\sin 2x$



reflection
 $y = -3 \sin 2(x-0) + 0$

Amplitude = $|-3|$
 $= 3$

Period = $\frac{2\pi}{b}$
 $= \frac{2\pi}{2}$
 $= \pi$

Starting point still at $x=0$



$d=0$ median line is at $y=0$

We could also use mapping.

(x, y)	$(\frac{1}{2}x, -3y)$
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5 key points

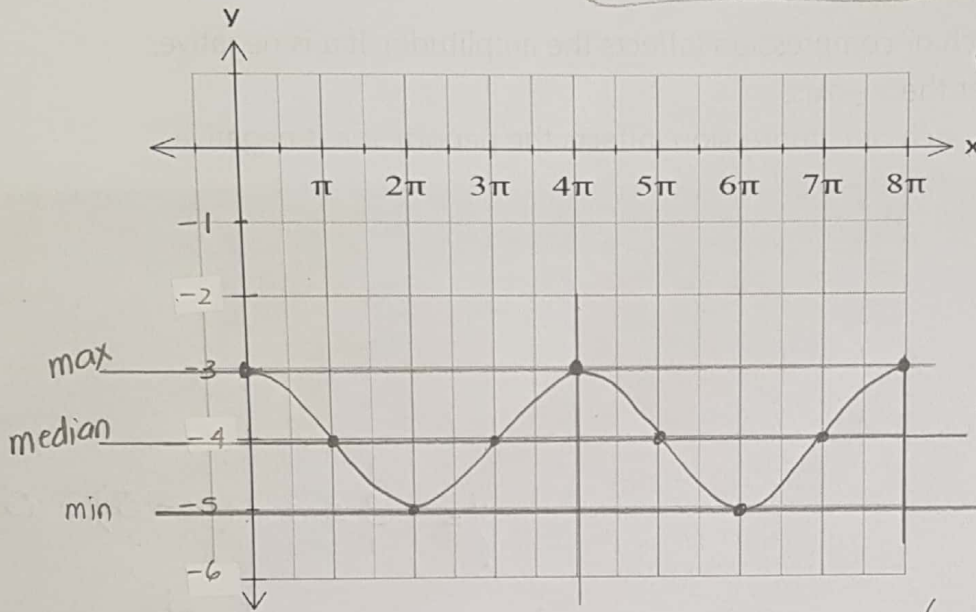
$(0, 0)$	$(0, 0)$
$(\frac{\pi}{2}, 1)$	$(\frac{\pi}{4}, -3)$
$(\pi, 0)$	$(\frac{\pi}{2}, 0)$
$(\frac{3\pi}{2}, -1)$	$(\frac{3\pi}{4}, 3)$
$(2\pi, 0)$	$(\pi, 0)$

$a = -3$
 $b = 2$
 $c = 0$
 $d = 0$

$(\frac{1}{b}x + c, ay + d)$
 $(\frac{1}{2}x + 0, -3y, 0)$

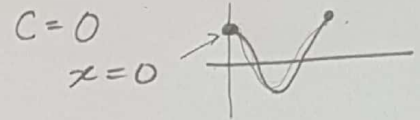
b) $y = \cos \frac{1}{2}x - 4$

$y = 1 \cos \frac{1}{2}(x - 0) - 4$



✓ Amplitude = 1

$b = \frac{1}{2}$ $P = \frac{2\pi}{\frac{1}{2}} = 4\pi$



✓ $d = -4$ median line is at $y = -4$

MAPPING

$(x, y) \rightarrow (\frac{1}{2}x + 0, |y - 4)$

(x, y)	$(2x, y - 4)$
$(0, 1)$	$(0, -3)$
$(\frac{\pi}{2}, 0)$	$(\pi, -4)$
$(\pi, -1)$	$(2\pi, -5)$
$(\frac{3\pi}{2}, 0)$	$(3\pi, -4)$
$(2\pi, 1)$	$(4\pi, -3)$

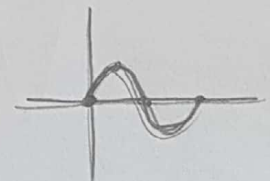
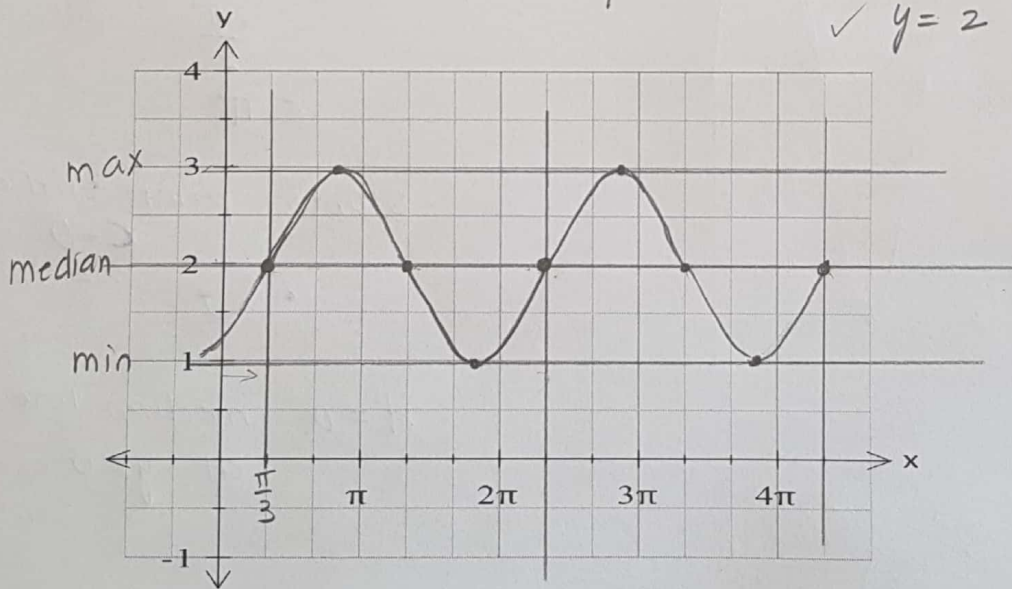
c) $y = |\sin(x - \frac{\pi}{3})| + 2$

✓ amp = 1

Period = $\frac{2\pi}{1} = 2\pi$

phase shift = $\frac{\pi}{3}$

median line
✓ $y = 2$



(x, y)	$(2x, y - 4)$
$(0, 1)$	$(0, -3)$
$(\frac{\pi}{2}, 0)$	$(\pi, -4)$
$(\pi, -1)$	$(2\pi, -5)$
$(\frac{3\pi}{2}, 0)$	$(3\pi, -4)$
$(2\pi, 1)$	$(4\pi, -3)$

$$d) y = \frac{1}{2} \cos 2 \left(x - \frac{\pi}{6} \right) + 3$$

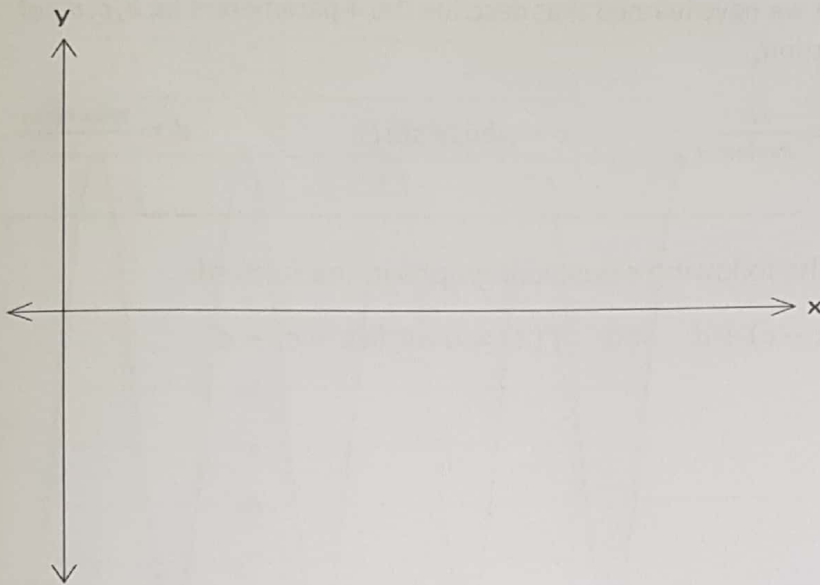
a
b
c
d

$$\text{amp} = \frac{1}{2}$$

$$P = \frac{2\pi}{2} = \pi \quad \text{shift} = \frac{\pi}{6}$$

$$\text{median} = 3$$

$$y = 3$$



* graph on graph paper

$$e) y = 2 \sin \frac{1}{2} (x + \pi) + 1$$

$$a = 2 \quad \text{amplitude}$$

$$b = \frac{1}{2} \quad P = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$\text{phase shift} = -\pi$$

$$\text{median} = 1$$

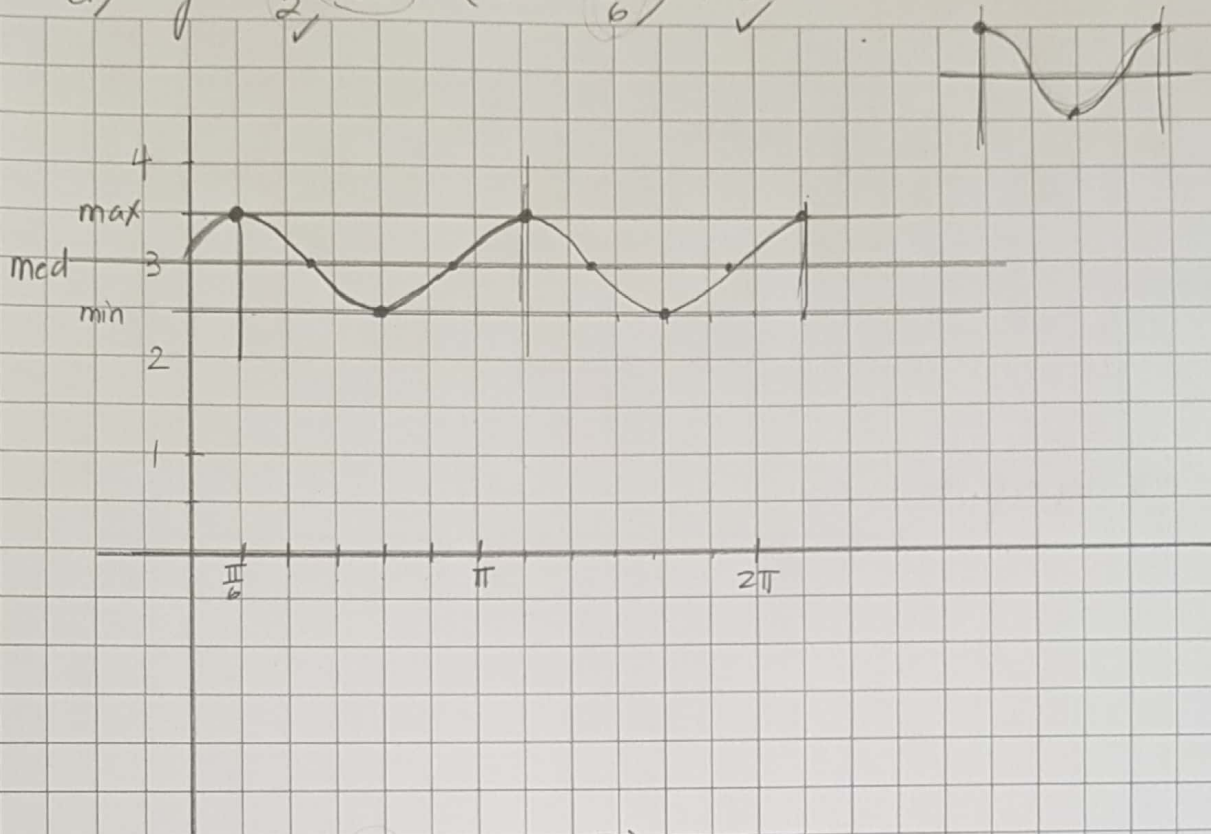
$$y = 1$$



* graph on graph paper

Assignment Time! Work on p.534- 3-5, 8-11, MC 1&2

$$d) y = \frac{1}{2} \cos 2 \left(x - \frac{\pi}{6} \right) + 3$$



$$e) y = 2 \sin \frac{1}{2} (x + \pi) + 1$$

$$p = 4\pi$$

