

Lesson 4: Determine equations given the graphs of $y = \sin x$ and $y = \cos x$

Note the following formulas that we have learned that describe the 4 parameters (a, b, c, d) of the equation of a sinusoidal function.

$$a = \frac{|\max - \min|}{2}$$

$$b = \frac{2\pi}{\text{Period}}$$

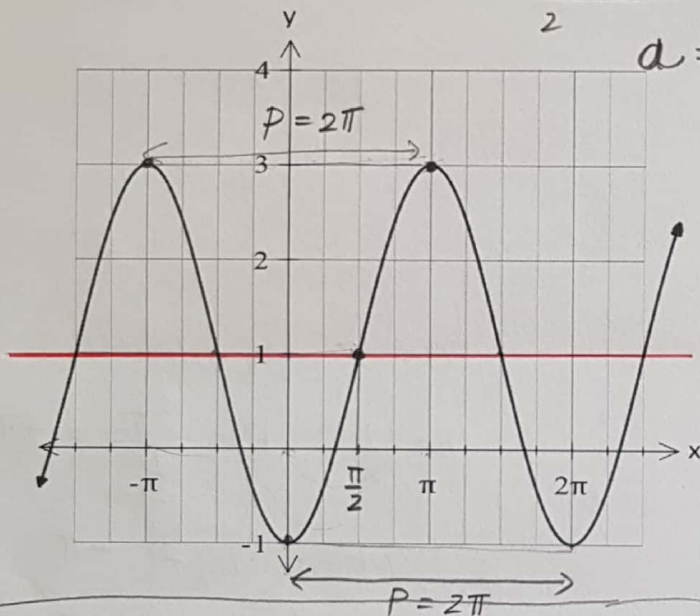
$c = \text{phase shift}$

$$d = \frac{\max + \min}{2}$$

Determine the equations for the following sinusoidal graphs in the form of:

$$f(x) = a \sin b(x - c) + d \quad \text{and} \quad f(x) = a \cos b(x - c) + d$$

Example 1



$$\text{amp} = \frac{|\max - \min|}{2} = \frac{|3 - (-1)|}{2}$$

$$a = 2 \checkmark$$

$$d = \frac{\max + \min}{2}$$

$$d = \frac{3 + (-1)}{2}$$

$$d = 1 \checkmark$$

$$b = \frac{2\pi}{P}$$

$$b = \frac{2\pi}{2\pi}$$

$$b = 1 \checkmark$$

$c \Rightarrow$ where the sine or cosine starts normally (phase shift)

For sine, phase shift is shifted $\frac{\pi}{2}$ to the right.

$$y = 2 \sin 1 \left(x - \frac{\pi}{2} \right) + 1$$

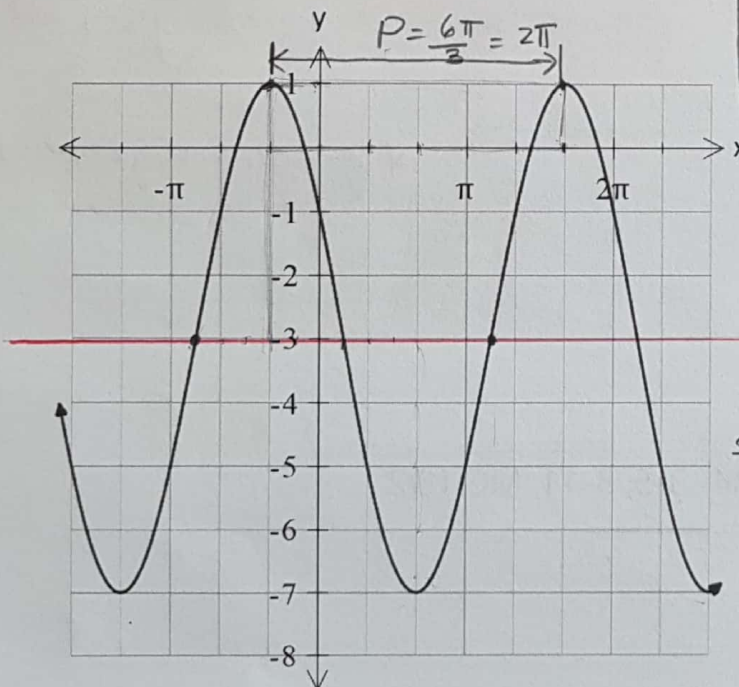
For cosine, the phase shift is shifted π to the right

$$y = 2 \cos 1 (x - \pi) + 1 \quad \text{OR}$$

$$y = 2 \cos 1 (x - -\pi) + 1$$

$$y = 2 \cos 1 (x + \pi) + 1$$

Example 2



$$a = \frac{1 - (-7)}{2}$$

$$a = 4$$

$$d = \frac{1 + (-7)}{2}$$

$$d = -3 \quad (\text{median line})$$

$$\text{Period} = 2\pi$$

$$\therefore b = \frac{2\pi}{2\pi} = 1$$

SINE: phase shift $\frac{7\pi}{6}$ right.

$$y = 4 \sin 1 \left(x - \frac{7\pi}{6} \right) - 3$$

Phase shift $\frac{5\pi}{6}$ left

$$y = 4 \sin 1 \left(x - -\frac{5\pi}{6} \right) - 3$$

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Example 2) other possible equation for the given graph.

For cosine: phase shift of $\frac{5\pi}{3}$ right

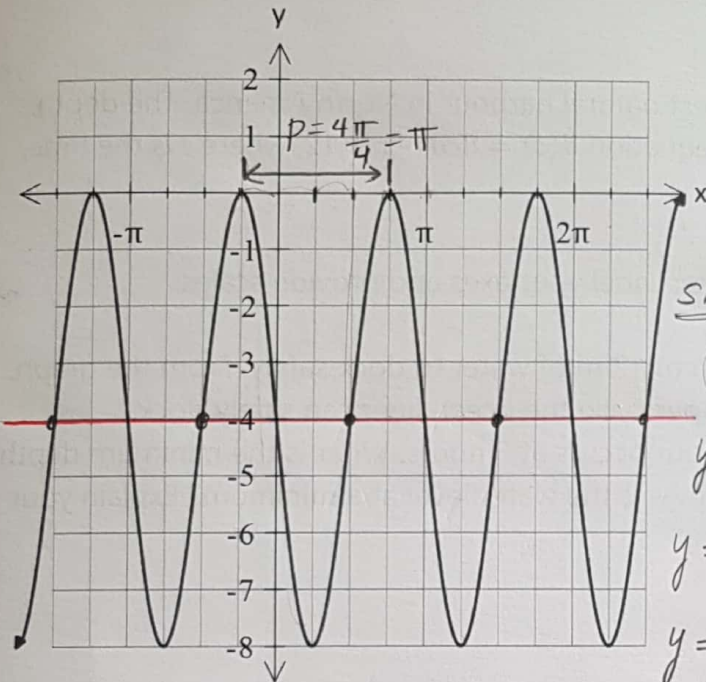
$$y = 4 \cos \left(x - \frac{5\pi}{3} \right) - 3$$

other possibility: for cosine: phase shift $\frac{\pi}{3}$ to the left.

$$y = 4 \cos \left(x - -\frac{\pi}{3} \right) - 3$$

$$y = 4 \cos \left(x + \frac{\pi}{3} \right) - 3$$

Example 3



$$a = \frac{0 - (-8)}{2} = 4 \checkmark$$

$$d = \frac{0 + (-8)}{2} = -4 \checkmark \text{ (median line)}$$

$$\text{Period} = \pi$$

$$b = \frac{2\pi}{\pi} = 2 \checkmark$$

SINE: PHASE SHIFTS: that are possible.

$$\frac{2\pi}{4}, \frac{6\pi}{4}, -\frac{2\pi}{4}, -\frac{6\pi}{4}$$

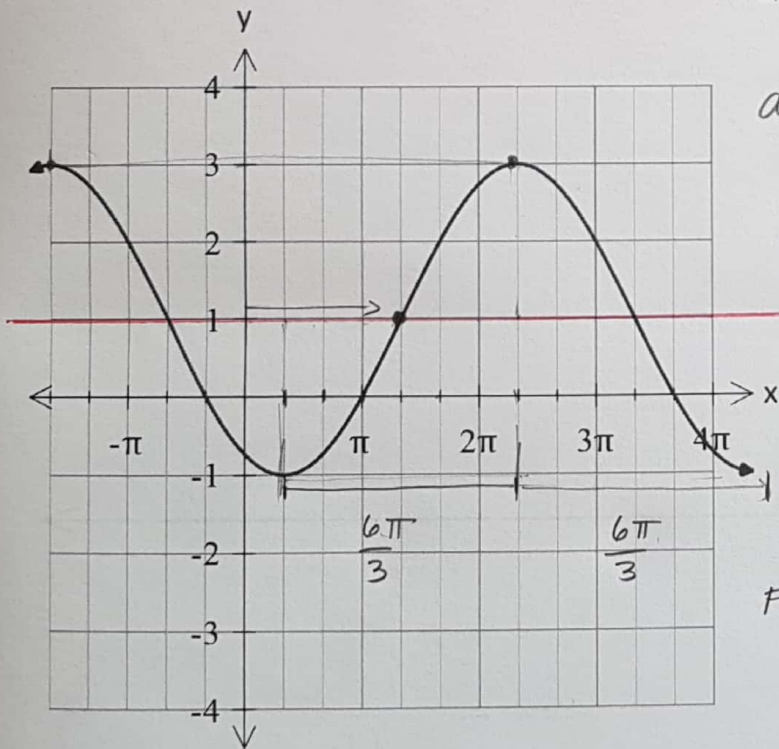
$$y = 4 \sin\left(2\left(x - \frac{2\pi}{4}\right)\right) - 4$$

$$y = 4 \sin\left(2\left(x - \frac{6\pi}{4}\right)\right) - 4$$

$$y = 4 \sin\left(2\left(x - -\frac{2\pi}{4}\right)\right) - 4$$

$$y = 4 \sin\left(2\left(x - -\frac{6\pi}{4}\right)\right) - 4$$

Example 4



$$a = \frac{3 - (-1)}{2} = 2 \checkmark$$

$$d = \frac{3 + (-1)}{2} = 1 \checkmark$$

$$P = \frac{12\pi}{3} = 4\pi$$

$$b = \frac{2\pi}{4\pi} = \frac{1}{2} \checkmark$$

FOR SINE: possible shifts.

$$\frac{4\pi}{3}$$

$$y = 2 \sin \frac{1}{2} \left(x - \frac{4\pi}{3}\right) + 1$$

FOR COSINE: possible phase shift

$$\frac{7\pi}{3}$$

$$y = 2 \cos \frac{1}{2} \left(x - \frac{7\pi}{3}\right) + 1$$

For practice: pg. 535 Q 6, 7

Test on CH6
Part 2 Graphing
trig function.
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Example 3) Other possible equations for the given graph.

For cosine: possible phase shifts.

$$-\frac{5\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$y = 4 \cos 2 \left(x - \frac{-5\pi}{4} \right) - 4$$

$$y = 4 \cos 2 \left(x - \frac{-\pi}{4} \right) - 4$$

$$y = 4 \cos 2 \left(x + \frac{3\pi}{4} \right) - 4$$

$$y = 4 \cos 2 \left(x + \frac{7\pi}{4} \right) - 4$$