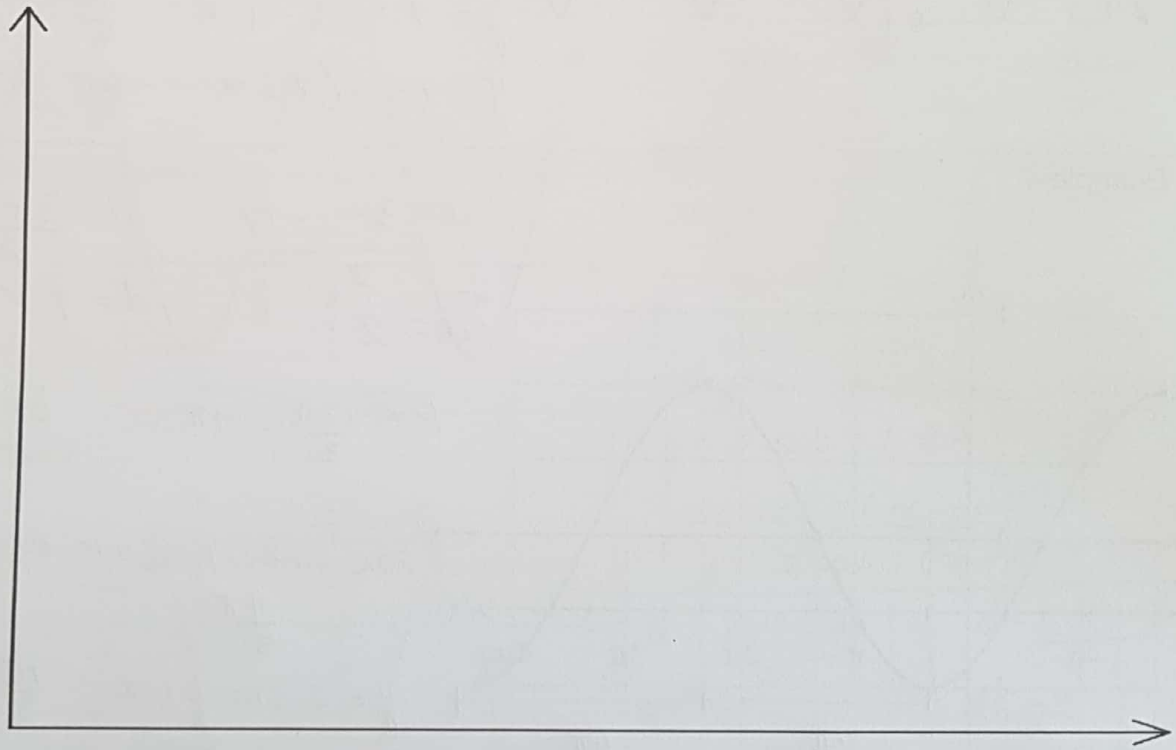
Lesson 5: Determine a trigonometric function to model and solve a problem

Example 1: A town in BC has the deepest natural harbour in North America. The depth, d, in metres, can be measured by the equation $d(t) = 8cos\frac{\pi}{6}t + 12$, where t is the time, in hours, after the first high tide.

a) Sketch the function for two cycles. Label your axes and provide scales.
b) What is the period of the tide? $P = \frac{2\pi}{\pi/6} = 12$ hours
c) An ocean liner needs a minimum of 12 m of water to dock safely. From the graph, determine the number of hours per cycle the ocean liner can safely dock. 6 hours.
d) The minimum depth of the harbour occurs at 6 hours. What is the minimum depth of the harbour? At what other times is the water level at a minimum? Explain your solution. minimum depth is 4 meters and it happens at 6 hours and 18 hours from the high tide.



a) $d = 8\cos\frac{\pi}{6}(t-0) + 12$

Graph on graph paper

median = 12

amp = $|8| = 8$

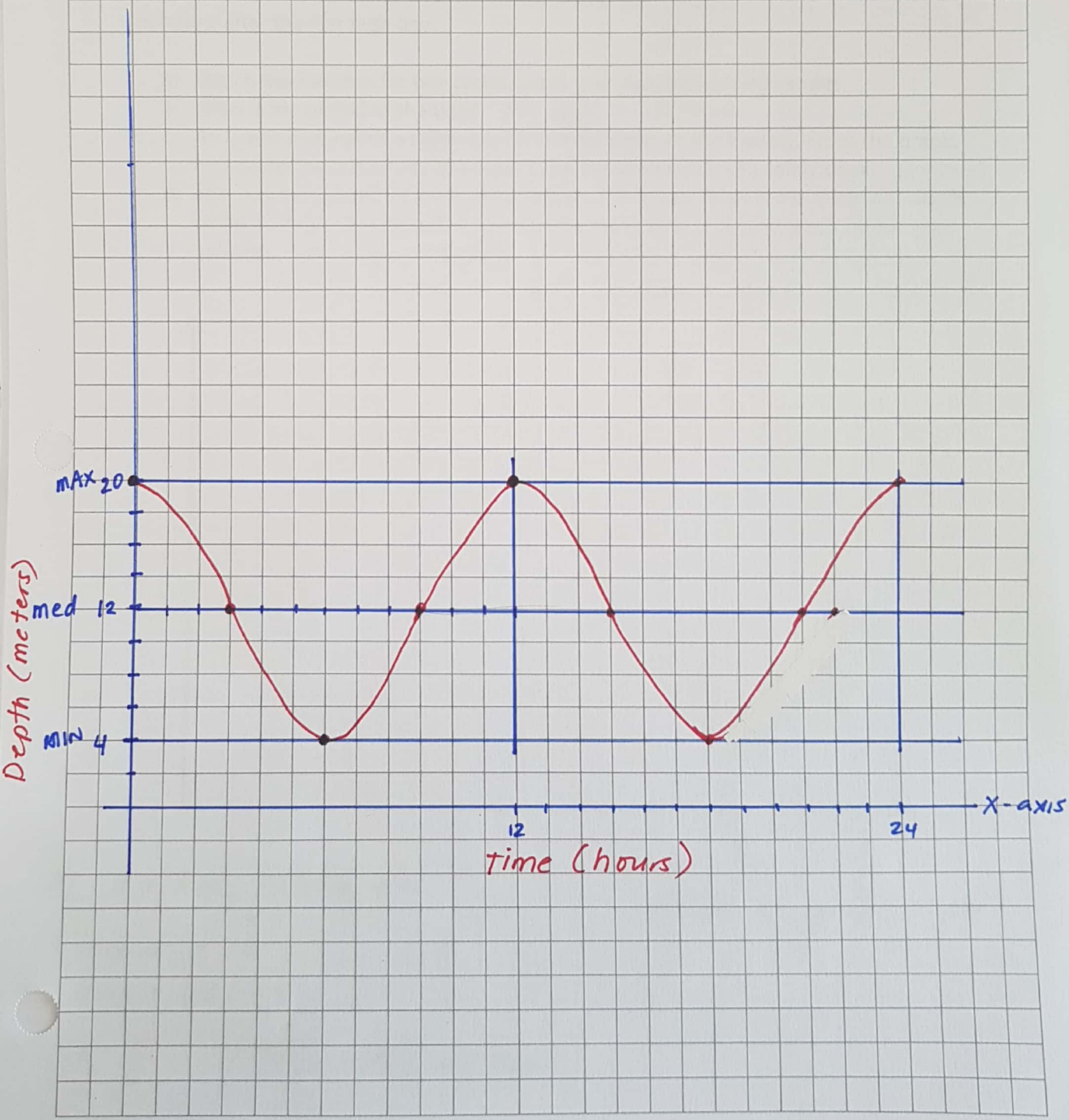
$b = \frac{\pi}{6}$ $P = \frac{2\pi}{\frac{\pi}{6}} = 2\pi\left(\frac{6}{\pi}\right) = 12$ hours.

Ex 1) $d(t) = 8\cos\frac{\pi}{6}(t-0)+12$

med = 12

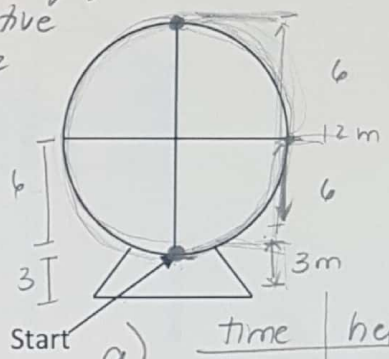
max = 12 + 8 = 20

min = 12 − 8 = 4



Example 2: The height of a Ferris wheel above ground follows a sinusoidal path over time. Its starting point is at the minimum height, 3 m above ground. The diameter of the Ferris wheel is 12 m. It takes 16 seconds to make one full trip around.

a) Sketch the function for two full cycles. Label your axes and provide scales.
b) Determine an equation of this sinusoid.
c) Determine the height of the Ferris wheel above ground 12 seconds after its start. 9 meters
d) What is the period of this function? What does the period represent in this context? $P = 16$ seconds; and it is the time it took for one complete cycle of ferris wheel.
e) State a limitation on the domain of this function.
↳ Domain has to be positive and it ends when the cycle stops.



a)

time	height
0	3
4	9
8	15
12	9
16	3

b)

$a = \frac{max - min}{2}$

$a = \frac{15 - 3}{2}$

$a = 6$ meters ✓

$d = \frac{max + min}{2}$

$d = \frac{15 + 3}{2}$

$d = 9$ meters ✓

$P = 16$, $b = \frac{2\pi}{P} = \frac{2\pi}{16} = \frac{\pi}{8}$ ✓

For sine, phase shift is 4
∴ the equation is $h = 6 \sin \frac{\pi}{8}(t - 4) + 9$

For cosine, phase shift is 8
∴ the equation is $h = 6 \sin \frac{\pi}{8}(t - 8) + 9$

Assignment Time! Work on p.544 Check Your Understanding 1a;

p.548- 4-6, 8b