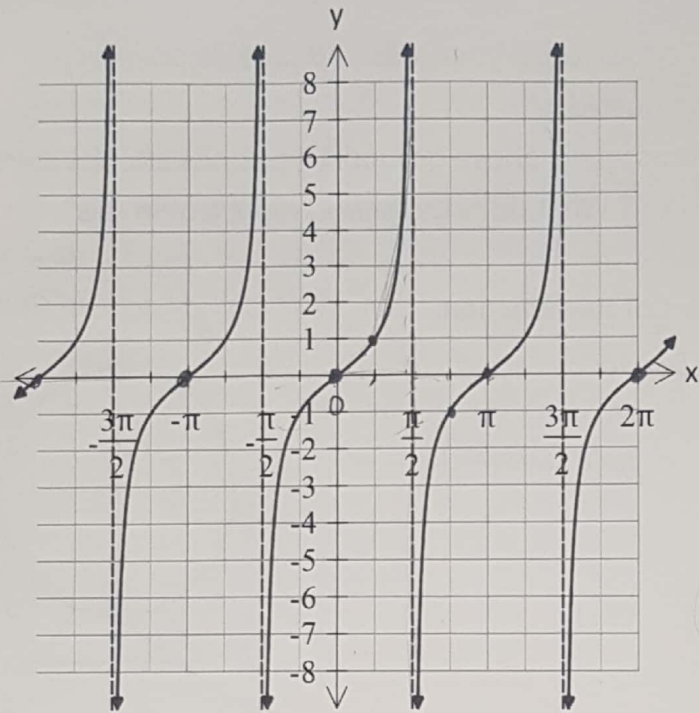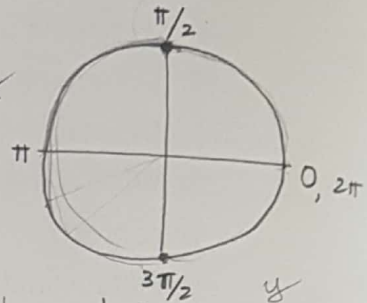## Lesson 6: The tangent function.

The graph of the tangent function $y = \tan x$ is periodic but not sinusoidal.



$y = \tan x$

$y = \dfrac{\sin x}{\cos x}$

| Angle $\theta$ | $\sin \theta$ | $\cos \theta$ | $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$ |
|---|---|---|---|
| $0$ | $0$ | $1$ | $0$ |
| $\dfrac{\pi}{6}$ | $\dfrac{1}{2}$ | $\dfrac{\sqrt{3}}{2}$ | $\dfrac{1}{2} \div \dfrac{\sqrt{3}}{2} = \dfrac{1}{\sqrt{3}}$ |
| $\dfrac{\pi}{4}$ | $\dfrac{\sqrt{2}}{\sqrt{2}}$ | $\dfrac{\sqrt{2}}{2}$ | $1$ |
| $\dfrac{\pi}{3}$ | $\dfrac{\sqrt{3}}{2}$ | $\dfrac{1}{2}$ | $\dfrac{\sqrt{3}}{2} \div \dfrac{1}{2} = \sqrt{3}$ |
| $\dfrac{\pi}{2}$ | $1$ | $0$ | undefined |
| $\dfrac{2\pi}{3}$ | $\dfrac{\sqrt{3}}{2}$ | $-\dfrac{1}{2}$ | $\dfrac{\sqrt{3}}{2} \div -\dfrac{1}{2} = -\sqrt{3}$ |
| $\dfrac{3\pi}{4}$ | $\dfrac{\sqrt{2}}{2}$ | $-\dfrac{\sqrt{2}}{2}$ | $-1$ |
| $\dfrac{5\pi}{6}$ | $\dfrac{1}{2}$ | $-\dfrac{\sqrt{3}}{2}$ | $-\dfrac{1}{\sqrt{3}}$ |
| $\pi$ | $0$ | $-1$ | $0$ |

Characteristics of $y = \tan x$:

Period: $\pi$

Asymptotes happen when $\cos \theta = 0$

Equation(s) of asymptotes:

$\dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \ldots$

general Equation for asymptote $\dfrac{\pi}{2} + \pi(k)$, where $k \in \mathbb{Z}$

Range: $(-\infty, \infty)$ or $y \in \mathbb{R}$

$y$-intercept: $y = 0$

$x$-intercepts:
$\ldots, -\pi, 0, \pi, 2\pi, \ldots$

$\pi(k)$, where $k \in \mathbb{Z}$

Notes:
- The function $y = \tan x$ has no maximum or minimum values.
- The locations of the asymptotes correspond to angles where $\tan x$ is undefined on the unit circle. $y = \tan x$

$y = \dfrac{\sin x}{\cos x}$ ← asymptote when $\cos x = 0$