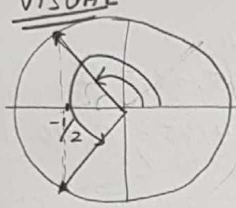
Lesson 1: Solving First- and Second-Degree Trigonometric Equations

You have already explored solving trig equations in both degrees and radians. This lesson will expand on the concepts you learned in Chapter 6 Part 1.

Example 1 (Review of Chapter 6):

Solve the equation $\cos\theta = -\frac{1}{2}$ over the domain $\boxed{0° \le \theta < 360°}$.

VISUAL



* Where is $\cos\theta$ -'ve?

S	A
T	C

In QII and QIII.

$\cos\theta_R = \frac{1}{2}$

$\theta_R = \cos^{-1}\left(\frac{1}{2}\right)$

$\boxed{\theta_R = 60°}$

Therefore θ in QII: $\theta = 180 - 60°$
$= 120°$

QIII: $\theta = 180 + 60$
$= 240°$

CHECK:

$\cos 120 = -\frac{1}{2}$

$-\frac{1}{2} = -\frac{1}{2}$

$\cos 240 = -\frac{1}{2}$

$-\frac{1}{2} = -\frac{1}{2}$

When solving a first-degree trig equation, isolate the trig function so that the equation is in the form $\sin\theta = a$, $\cos\theta = a$, or $\tan\theta = a$, where a is a constant. Give exact solutions wherever possible.

Example 2: Determine Exact Roots of a Trigonometric Equation

a) Solve the equation $\sqrt{2}\sin x - 3 = -2$ for $\boxed{0° \le x < 360°}$.

$\sqrt{2}\sin x - 3 = -2$

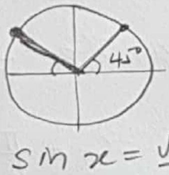
$\sqrt{2}\sin x = -2 + 3$

$\sqrt{2}\sin x = 1$

$\sin x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\sin x = \frac{\sqrt{2}}{2}$

Where is $\sin x$ +'ve?
in QI and II



$\sin x = \frac{\sqrt{2}}{2}$

$x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) \Rightarrow x_R = 45°$

Therefore
x in QI: $x = 45°$ ✓

x in QII: $x = 180 - 45°$
$= 135°$ ✓

b) Solve the equation $5\tan\theta + \sqrt{3} = 2\tan\theta$ for $\boxed{-2\pi < \theta \le 0.}$

$5\tan\theta + \sqrt{3} = 2\tan\theta$

$5\tan\theta - 2\tan\theta = -\sqrt{3}$

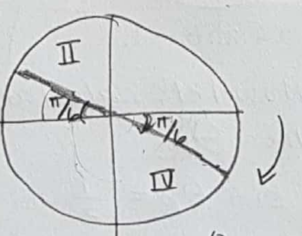
$3\tan\theta = -\sqrt{3}$

$\tan\theta = -\frac{\sqrt{3}}{3}$

Where is $\tan\theta$ -'ve?
In QII and QIV

$\tan\theta_R = \frac{\sqrt{3}}{3}$

$\theta_R = \frac{\pi}{6}$



In QIV: $\theta = -\frac{\pi}{6}$

In QII: $\theta = -\pi - \frac{\pi}{6}$
$= -\frac{7\pi}{6}$

CHECKING

$5\tan(-\pi/6) + \sqrt{3} = 2\tan(-\frac{\pi}{6})$

$-1.1547 = -1.1547$ ✓

$5\tan\left(-\frac{7\pi}{6}\right) + \sqrt{3} = 2\tan\left(-\frac{7\pi}{6}\right)$

$-1.1547 = -1.1547$ ✓

the angle is not the special angles

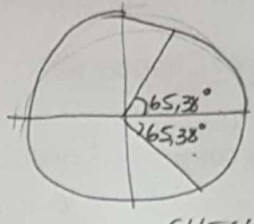
Example 3: Determining Approximate Roots of a Trigonometric Equation

a) Solve the equation $5\sec x - 4 = 8$ for $-180° \le x < 180°$. $[-180°, 0] \cup [0, 180°]$

Where is $\cos x$ +'ve?
in QI and QIV

$5\sec x - 4 = 8$
$5\sec x = 12$
$\sec x = \frac{12}{5}$
$\cos x = \frac{5}{12}$

$\cos x_R = \frac{5}{12}$
$x_R = \cos^{-1}\left(\frac{5}{12}\right)$
$x_R = 65.38°$



In QI: $x = 65.38°$
In QIV: $x = -65.38°$

CHECKING
$\cos 65.38 = \frac{5}{12}$
$0.4165 = 0.416$ ✓

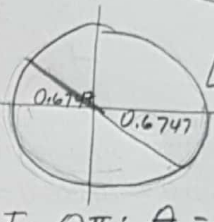
$\cos -65.38 = \frac{5}{12}$
$0.4165 = 0.416$ ✓

b) Solve the equation $5 - 3\tan\theta = 2\tan\theta + 9$ for $\left[-\pi, \frac{3\pi}{2}\right)$.

where is $\tan\theta$ -'ve?
in QII and QIV.

$5 - 3\tan\theta = 2\tan\theta + 9$
$-3\tan\theta - 2\tan\theta = 9 - 5$
$-5\tan\theta = 4$
$\tan\theta = \frac{4}{-5}$

Now calculate Ref. angle.
$\tan\theta_R = \frac{4}{5}$
$\theta_R = \tan^{-1}\left(\frac{4}{5}\right)$
$\theta_R = 0.6747^r$



$\left[-\pi, \frac{3\pi}{2}\right]$
$[-\pi, 0] \cup [0\ \frac{3\pi}{2}]$

In QII: $\theta = \pi - 0.6747 = 2.47^r$
In QIV: $\theta = -0.67^r$

There are an *infinite* number of solutions to any trigonometric equation that has solutions – writing ALL of the solutions is known as the "general solution".

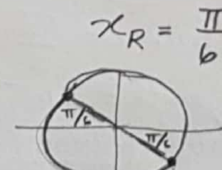
Example 4: Determining the General Solution of a Trigonometric Equation

a) Solve the equation $\cot x + \sqrt{3} = 0$ for $0 \le x < 2\pi$, then write the general solution.

$\cot x + \sqrt{3} = 0$
$\cot x = -\sqrt{3}$
$\tan x = -\frac{1}{\sqrt{3}}$

tan is -'ve in QII & IV

$\tan x_R = \frac{1}{\sqrt{3}}$
$x_R = \frac{\pi}{6}$



Therefore
x in QII: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

x in QIV: $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

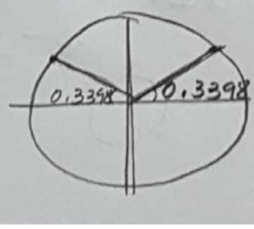
The general solution
$\frac{5\pi}{6} + \pi k,\ k \in \mathbb{Z}$

b) Solve the equation $7\sin\theta - 2 = 4\sin\theta - 1$.

$7\sin\theta - 2 = 4\sin\theta - 1$
$7\sin\theta - 4\sin\theta = -1 + 2$
$3\sin\theta = 1$
$\sin\theta = \frac{1}{3}$

Where is $\sin\theta$ +'ve?
in QI and QII.

Now let's calculate the θ_R.
$\sin\theta_R = \frac{1}{3}$
$\theta_R = \sin^{-1}\left(\frac{1}{3}\right)$
$\theta_R = 0.3398^r$



θ in QI: $\theta = 0.3398^r$

θ in QII: $\theta = \pi - 0.3398^r$
$= 2.8018^r$

The general solution
$0.3398^r + 2\pi k,\ k \in \mathbb{Z}$
AND
$2.8018^r + 2\pi k,\ k \in \mathbb{Z}$