pg 544 Check your understanding

1) Period = 20 seconds ; therefore $b = \frac{2\pi}{p} = \frac{2\pi}{20} = \frac{\pi}{10}$

min = 15cm
max = 35cm

We can calculate the amplitude and median line.

$$a = \frac{|max - min|}{2}$$

$$a = \frac{35 - 15}{2}$$

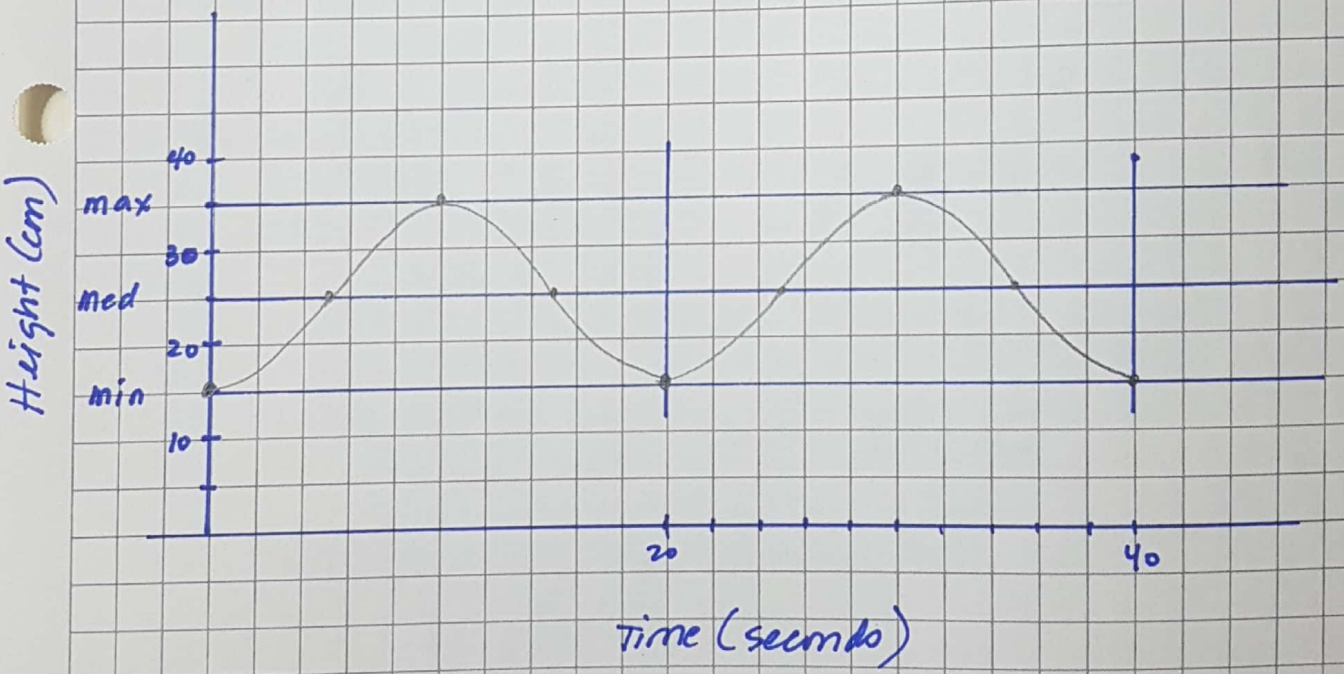
$$a = 10$$

$$d = \frac{max + min}{2}$$

$$d = \frac{35 + 15}{2}$$

$$d = \frac{50}{2} = 25$$

* We can graph, and then we can determine the phase shift.



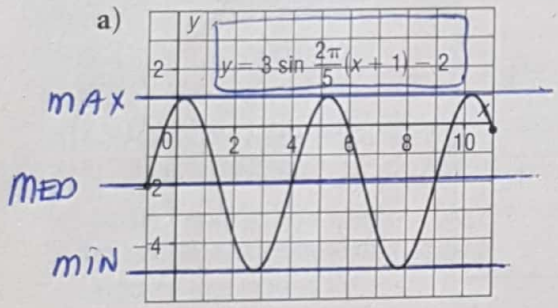
Exercises

A

3. Identify the transformations that would be applied to the graph of $y = \cos x$ to get the graph of $y = \frac{3}{4}\cos\frac{\pi}{5}(x + 3) - \frac{3}{2}$.

4. Identify the following characteristics of each graph below: amplitude; period; phase shift; equation of the centre line; domain; maximum value; minimum value; range

a)



$a = \frac{1-(-5)}{2}$ $\quad d = \frac{1+(-5)}{2}$

$a = 3$ $\quad d = -2$

$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{5}} = 5$

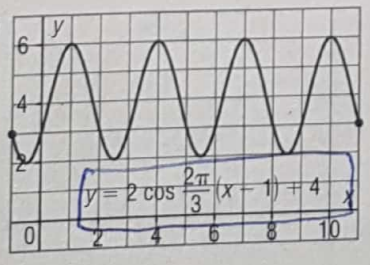
max = 1

phase shift = −1

min = −5

range = [−5, 1] $\quad$ Domain: [−1, 11]

b)



max = 6

min = 2

$a = 2$

$d = +4$

} We can determine the amplitude and median line from the given equation

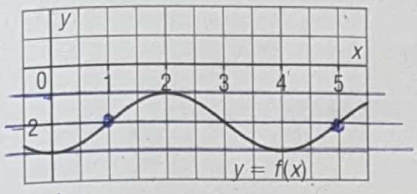
$\text{period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{3}} = 3$

Domain: [−1, 11]

Range: [2, 6]

B

5. a) For the function graphed below, identify the values of a, b, c, and d in $y = a \sin b(x - c) + d$, then write an equation for the function.



$P = 4$

max $= -1$

min $= -3$

$a = \frac{|-1 - -3|}{2}$

$a = 1$

$d = \frac{-1 + -3}{2}$

$d = -2$

period $= 4$

$b = \frac{2\pi}{4} = \frac{\pi}{2}$

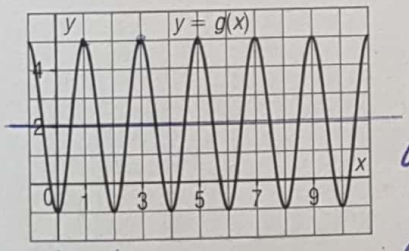
possible phase shift for sine function is 1 or 5

$y = 1 \sin \frac{\pi}{2}(x - 1) - 2$

or

$y = 1 \sin \frac{\pi}{2}(x - 5) - 2$

b) For the function graphed below, identify the values of a, b, c, and d in $y = a \cos b(x - c) + d$, then write an equation for the function.



$P = 2$

max $= 5$

min $= -1$

$a = \frac{5 - -1}{2}$

$a = 3$

$d = \frac{5 + -1}{2}$

$d = 2$

period $= 2$

$b = \frac{2\pi}{2}$

$b = \pi$

possible phase shift for cos are 1, 3, 5, 7, 9, ...

$y = 3 \cos \pi(x - 1) + 2$

OR

$y = 3 \cos \pi(x - 3) + 2$

OR

$y = 3 \cos \pi(x - 5) + 2$

$y = 3 \cos \pi(x - 7) + 2$

$y = 3 \cos \pi(x - 9) + 2$

6. Use transformations to sketch a graph of each function for $-5 \le x \le 5$.

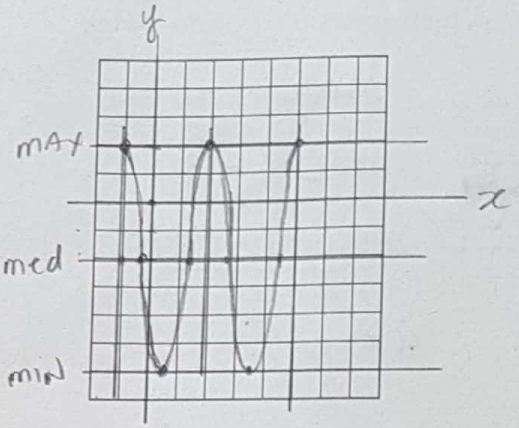
a) $y = 4\cos\frac{2\pi}{3}(x + 1) - 2$

Median line: $y = -2$

Amplitude = 4

Period = $\frac{2\pi}{\frac{2\pi}{3}} = 3$

Phase shift = -1



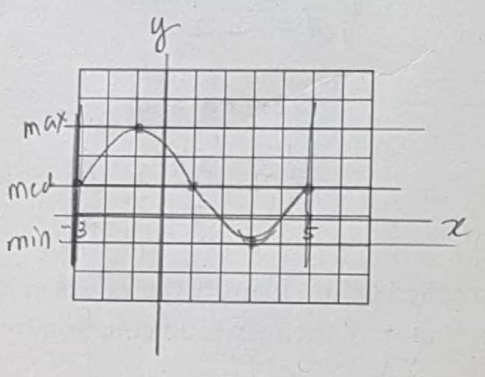
b) $y = 2\sin\frac{\pi}{4}(x + 3) + 1$

Median line: $y = 1$

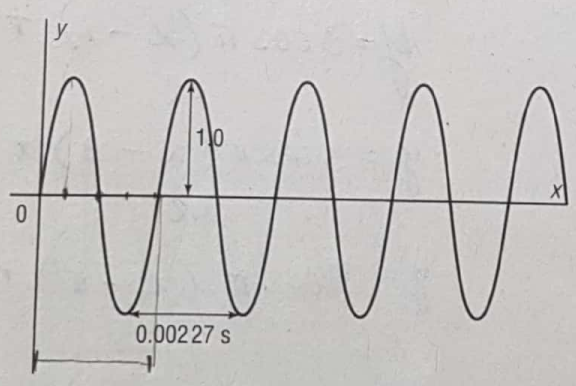
amp = 2

$P = \frac{2\pi}{\frac{\pi}{4}} = 2\pi\left(\frac{4}{\pi}\right) = 8$

Phase shift = -3



7. A sound wave is a sinusoidal curve, as illustrated by the oscilloscope image below. Determine the function that best describes this graph.



0.00227

amp = 1

period = 0.00227

$b = \frac{2\pi}{P} = \frac{2\pi}{0.00227} = 2767.92$

No phase shift $C = 0$

No vertical shift $d = 0$ (median line)

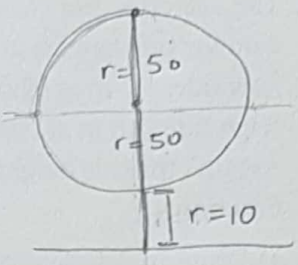
Therefore the function is

$y = 1\sin 2767.92(x - 0) + 0$

or

$y = 1\cos 2767.92(x - 0.0005675)$

8. A vertical wheel with radius 50 cm rotates about an axle that is 60 cm above the ground. A marker is placed at the top of the wheel. The wheel completes one rotation every 4 s.



a) i) Explain why a cosine function would be an appropriate model for the height, h centimetres, of the marker at any time t seconds.

This scenario relates the height of the marker as a function of time.

ii) For the graph of the cosine function from part i, identify the: period; phase shift; equation of the centre line; and amplitude. Explain how each characteristic relates to the conditions in the problem.

Time(s)	Height(cm)
0	110
1	60
2	10
3	60
4	110

period = 4 seconds $\Rightarrow b = \frac{2\pi}{4} = \frac{\pi}{2}$

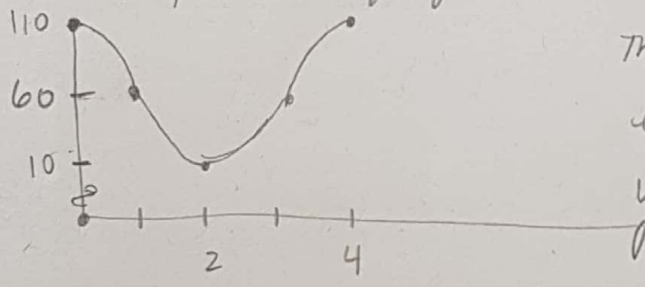
centre line = median line = $\frac{110+10}{2}$

$= 60$ cm

amplitude = $\frac{110-10}{2} = 50$ cm

b) Write an equation of a cosine function that models the motion of the wheel.

phase shift for cosine is 0



Therefore the cosine function equation is

$y = 50\cos\frac{\pi}{2}(x-0)+60$