THINK FURTHER

Write an equation in terms of cos x for the graph in *Example 2*.

Discuss the Ideas

1. Suppose you are given an equation of the form $y = a \sin b(x - c) + d$. Which characteristics of the graph can you identify?

2. What strategy would you use to sketch the graph of the transformation image of a sinusoidal function?

Exercises

A

3. Identify the transformations that would be applied to the graph of $y = \sin x$ to get the graph of $y = 10 \sin \frac{1}{3}(x - \pi) + 1$.

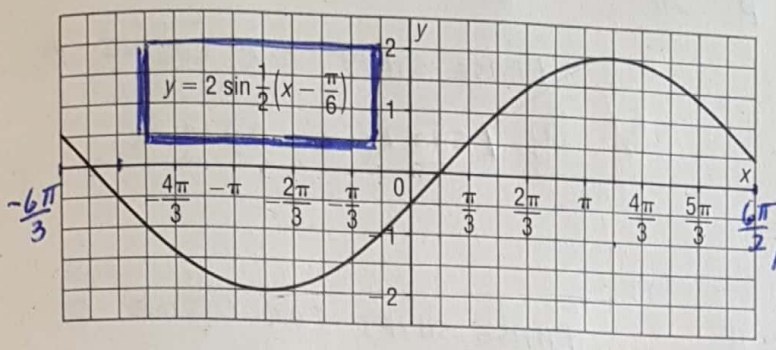
$a = 10$ Vertical stretch by a factor of 10

$b = \frac{1}{3}$ Horizontal stretch by a factor of $\frac{1}{\frac{1}{3}} = 3$

$c = \pi$ Horizontal translation to the right π unit.

$d = 1$ Vertical translation up 1 unit

4. Identify the following characteristics of the graph below: amplitude; period; phase shift; equation of the centre line; zeros; domain; maximum value; minimum value; range



amplitude $= \frac{\max - \min}{2}$

$= \frac{2-(-2)}{2}$

$a = 2$

median line $= \frac{2 + -2}{2}$

$y = 0$

phase shift: $x = \frac{\pi}{6}$

max = 2

min = −2

range: $[-2, 2]$ D: $[-2\pi, 2\pi]$

B

5. Use the given data to write an equation for each function.

a) a sine function with: amplitude 5; period 3π; equation of centre line $y = -2$; and phase shift $\frac{\pi}{3}$

$a = 5$ $\quad c = \frac{\pi}{3}$

$p = 3\pi$

$b = \frac{2\pi}{p} = \frac{2\pi}{3}$

$d = -2$

Sine function

$y = a\sin b(x - c) + d$

$y = 5\sin\frac{2\pi}{3}\left(x - \frac{\pi}{3}\right) + -2$

b) a cosine function with: maximum value 5; minimum value −2; period π; and phase shift $-\frac{\pi}{4}$

$a = \frac{\max - \min}{2}$

$a = \frac{5 - -2}{2}$

$a = 3,5$

$c = -\frac{\pi}{4}$

$p = \pi$

$b = \frac{2\pi}{p}$

$b = \frac{2\pi}{\pi}$

$b = 2$

No vertical shift, $d = 0$

cosine function

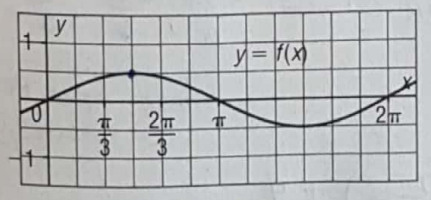
$y = a\cos b(x - c) + d$

$y = 3,5\cos 2\left(x - \frac{-\pi}{4}\right) + 0$

$y = 3,5\cos 2\left(x + \frac{\pi}{4}\right)$

6. Determine a possible equation for each function graphed below.

a)



PHASE SHIFT FOR SINE: 0

∴ EQUATION $y = a\sin b(x - c) + d$

$y = 0.5\sin 1(x - 0) + 0$

$a = 0.5$ $\quad p = 2\pi$

$d = 0$ $\quad b = \frac{2\pi}{p} = \frac{2\pi}{2\pi} = 1$

PHASE SHIFT FOR COSINE: $\pi/2$

∴ EQUATION $y = a\cos b(x - c) + d$

$y = 0.5\cos 1(x - \pi/2) + 0$

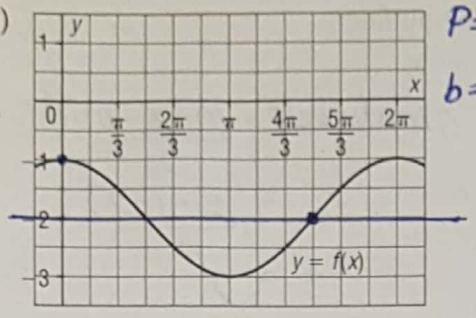
$a = \frac{-1 - -3}{2}$

$a = 1$

$d = \frac{-1 + -3}{2}$

$d = -2$

b)



$P = 2\pi$

$b = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$

PHASE SHIFT FOR SINE $\frac{9\pi}{6} = \frac{3\pi}{2}$

$y = 1 \sin 1\left(x - \frac{3\pi}{2}\right) - 2$

PHASE SHIFT FOR COSINE 0

$y = 1 \cos 1(x - 0) - 2$

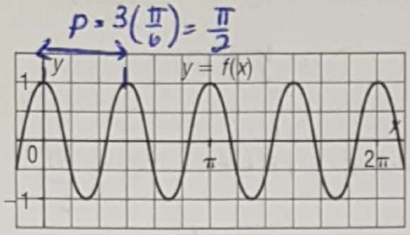
$a = \frac{1 - -1}{2}$

$a = 1$

$d = \frac{1 + -1}{2}$

$d = 0$

c)



$P = \frac{\pi}{2}$

$b = \frac{2\pi}{\frac{\pi}{2}} = 4$

PHASE SHIFT FOR COSINE 0

$y = 1 \cos 4(x - 0) + 0$

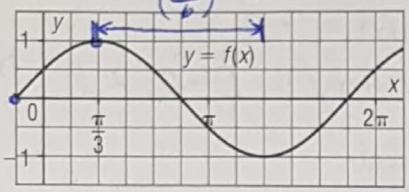
$a = \frac{1 - -1}{2}$

$a = 1$

$d = \frac{1 + -1}{2}$

$d = 0$

d)



$P = 2\pi$

$b = \frac{2\pi}{2\pi} = 1$

PHASE SHIFT FOR SINE $-\frac{\pi}{6}$

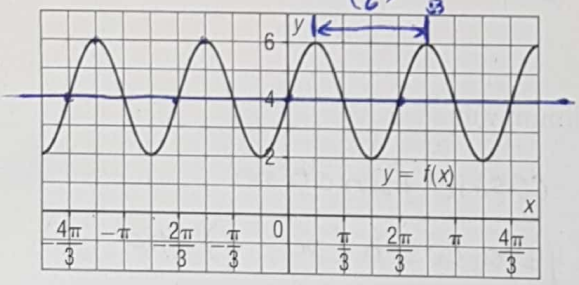
$y = 1 \sin 1\left(x - -\frac{\pi}{6}\right) + 0$

PHASE SHIFT FOR COSINE $\frac{\pi}{3}$

$y = 1 \cos 1\left(x - \frac{\pi}{3}\right) + 0$

7. a) For the function graphed below, identify the values of a, b, c, and d in $y = a \sin b(x - c) + d$, then write an equation for the function.

$4\left(\frac{\pi}{6}\right) = \frac{2\pi}{3}$



$a = \frac{6 - 2}{2}$

$a = 2$

$d = \frac{6 + 2}{2}$

$d = 4$

$P = \frac{2\pi}{3}$

$b = \frac{2\pi}{\frac{2\pi}{3}} = 3$

PHASE SHIFT FOR SINE: 0, or $\frac{2\pi}{3}, -\frac{2\pi}{3}, -\frac{4\pi}{3}$

THESE ARE POSSIBLE C-VALUES

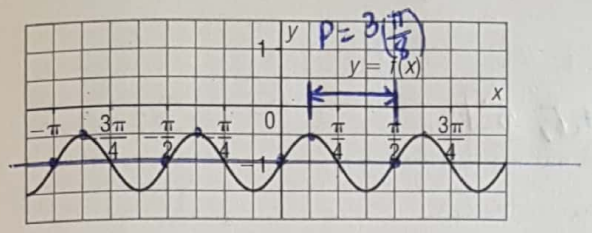
$y = 2 \sin 3(x - 0) + 4$

PHASE SHIFT FOR COSINE: $-\frac{7\pi}{6}, -\frac{3\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

THESE ARE POSSIBLE C-VALUES

$y = 2 \cos 3\left(x - -\frac{7\pi}{6}\right) + 4$

b) For the function graphed below, identify the values of a, b, c, and d in $y = a \cos b(x - c) + d$, then write an equation for the function.



$p = 3\left(\frac{\pi}{8}\right)$

$a = \frac{-0.5-(-1.5)}{2}$

$a = 0.5$

$d = \frac{-0.5+(-1.5)}{2}$

$d = -1$

$p = \frac{3\pi}{8}$

$b = \frac{2\pi}{\frac{3\pi}{8}}$

$b = 2\pi\left(\frac{8}{3\pi}\right) = \frac{16}{3}$

PHASE SHIFT FOR SINE: $-\pi, \frac{-\pi}{2}, 0, \frac{\pi}{2}$

THESE ARE POSSIBLE C-VALUES

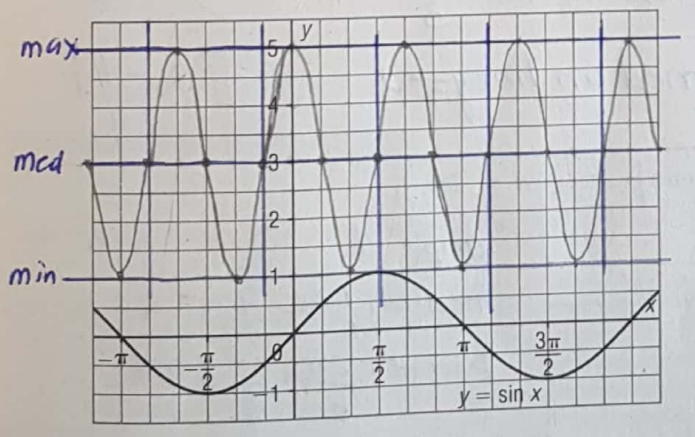
$y = 0.5 \sin \frac{16}{3}(x - -\pi) - 1$

PHASE SHIFTS FOR COSINE: $\frac{-7\pi}{8}, \frac{-3\pi}{8}, \frac{\pi}{8}, \frac{4\pi}{8}$

THESE ARE POSSIBLE C-VALUES

$y = 0.5 \cos \frac{16}{3}\left(x - \frac{-7\pi}{8}\right) - 1$

8. a) The graph of $y = \sin x$ is shown below. On the same grid, sketch the graph of $y = 2 \sin 3\left(x - \frac{\pi}{2}\right) + 3$. Describe your strategy.



$y = 2 \sin 3\left(x - \frac{\pi}{2}\right) + 3$

median line at 3

amplitude = 2

phase shift = $\frac{\pi}{2}$

Period = $\frac{2\pi}{b} = \frac{2\pi}{3} = \frac{4\pi}{6}$ ← since the scale is $\frac{\pi}{6}$, I used fractions w/ $\frac{\pi}{6}$

b) List the characteristics of the function $y = 2 \sin 3\left(x - \frac{\pi}{2}\right) + 3$.

amplitude = 2

period = $\frac{2\pi}{3}$

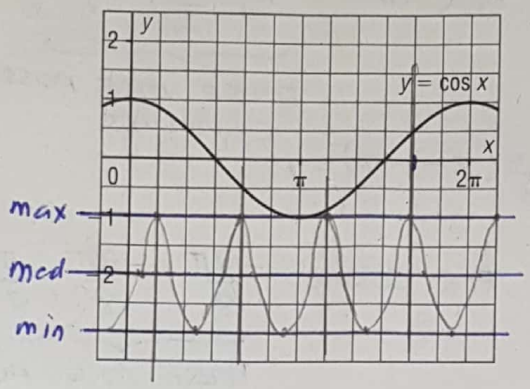
phase shift = $\frac{\pi}{2}$

median line at $y = 3$

D: $x \in \mathbb{R}$

R: $[1, 5]$

9. a) The graph of $y = \cos x$ is shown below. On the same grid, sketch the graph of $y = \cos 4\left(x + \frac{\pi}{3}\right) - 2$. Describe your strategy.



median = $-2 = y$

amp = 1

period = $\frac{2\pi}{4} = \frac{\pi}{2} \times \frac{3}{3} = \frac{3\pi}{6}$

phase shift = $-\frac{\pi}{3} \times \frac{2}{2} = \frac{-2\pi}{6}$

Since I cannot start at $-\frac{2\pi}{6}$, I could use the coterminal $-\frac{2\pi}{6} + 2\pi\left(\frac{6}{6}\right)$

$-\frac{2\pi}{6} + \frac{12\pi}{6}$

$\frac{10\pi}{6}$

b) List the characteristics of the function $y = \cos 4\left(x + \frac{\pi}{3}\right) - 2$.

amp = 1

period = $\frac{2\pi}{4} = \frac{\pi}{2}$

phase shift $-\frac{\pi}{3}$

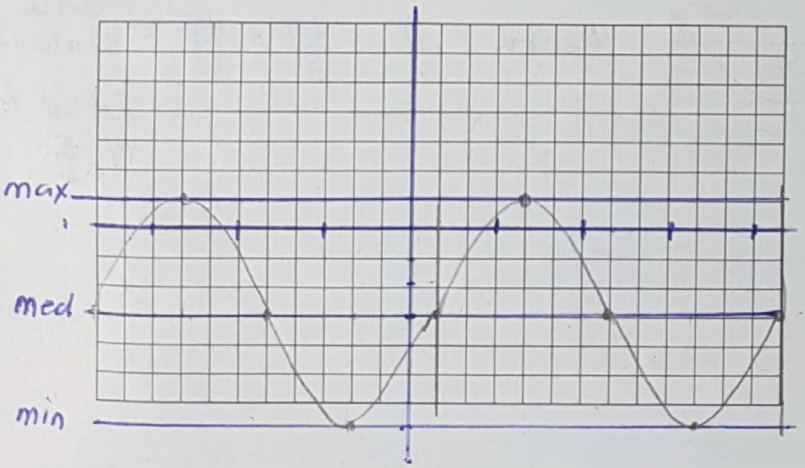
median line $y = -2$

D: $x \in \mathbb{R}$

R: $[-3, -1]$

10. Sketch the graph of each function for the domain $-2\pi \leq x \leq 2\pi$.

a) $y = 4 \sin \frac{1}{2}\left(x - \frac{\pi}{3}\right) - 3$



amp = 4

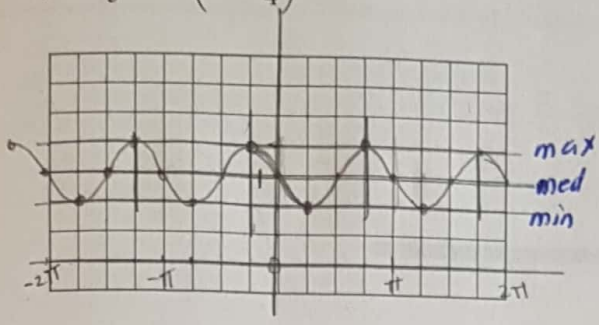
median line $y = -3$

Period = $\frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$

phase shift $\frac{\pi}{3}$

since phase shift is $\pi/3$, it would be good to have x-scale to be $\pi/3$

b) $y = \frac{1}{3}\cos 2\left(x + \frac{\pi}{4}\right) + 1$



amp $= \frac{1}{3}$

median $= 1$

period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift $= -\frac{\pi}{4}$

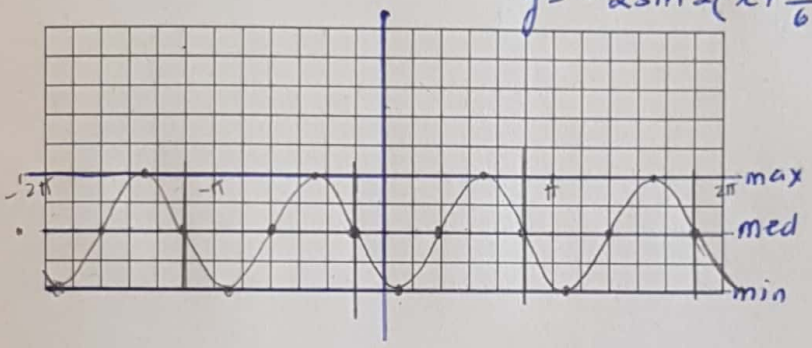
max $= 1 + \frac{1}{3} = 1\frac{1}{3}$

min $= 1 - \frac{1}{3} = \frac{2}{3}$

C

11. Use transformations to sketch the graph of $y = -2\sin\left(2x + \frac{\pi}{3}\right) - 2$ for $-2\pi \le x \le 2\pi$.

$y = -2\sin 2\left(x + \frac{\pi}{6}\right) - 2$



(*) There is a vertical reflection.

amp $= |-2| = 2$

median line $y = -2$

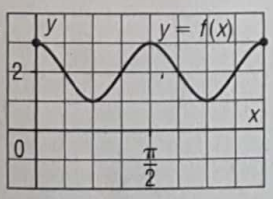
period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift $-\frac{\pi}{6}$

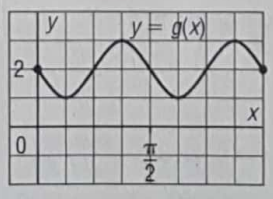
Multiple-Choice Questions

1. Which graph below can be described by $y = \cos 4\left(x + \frac{\pi}{8}\right) + 2$?

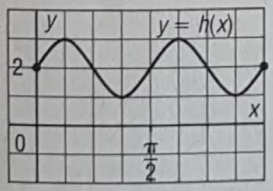
A.



B.



C.



D.