Pre-Calculus 40S Practice (Graphs of Trig Functions)

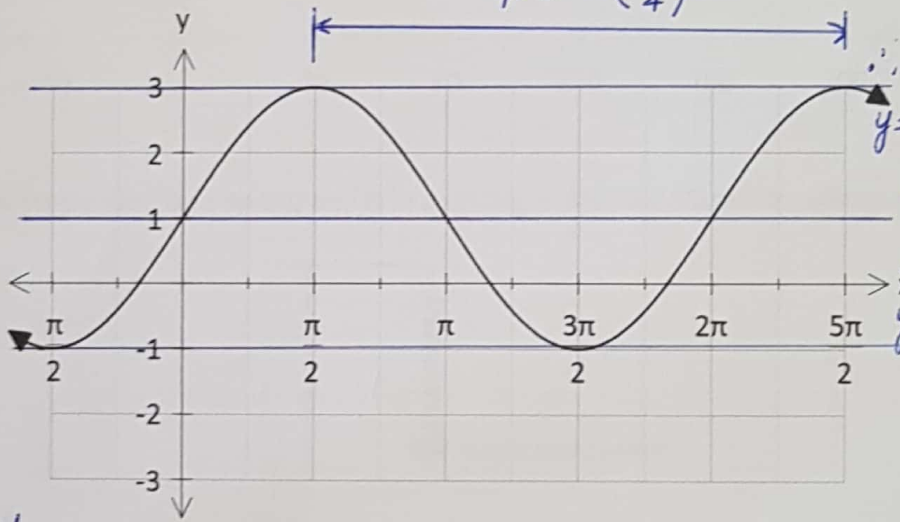
1. Sketch the graph of $y = 2\cos\frac{1}{2}(x) + 1$ from $0 \le x \le 2\pi$. ON GRAPH PAPER

2. Sketch the graph of $y = -\frac{1}{3}\sin\left(x - \frac{\pi}{4}\right) + 2$ from $-2\pi \le x \le 2\pi$. Then, state the amplitude, period, phase shift, and the equation of the median of this function. ON GRAPH PAPER

3. Sketch the graph of $y = \cos\pi(x - 3)$ from $0 \le x \le 4\pi$. ON GRAPH PAPER

4. Give equations of a sinusoidal function in terms of **BOTH** $\sin x$ and $\cos x$ that would match the graph given below:

$P = 8\left(\frac{\pi}{4}\right) = 2\pi$



$amp = \frac{|max - min|}{2}$

$= \frac{3-(-1)}{2}$

$a = 2$

$d = \frac{max + min}{2}$

$= \frac{3+(-1)}{2}$

$= 1$

$P = 2\pi$ $\quad b = \frac{2\pi}{2\pi} = 1$

FOR SINE, PHASE SHIFT IS $C = 0$

$\therefore$ SINE EQUATION.

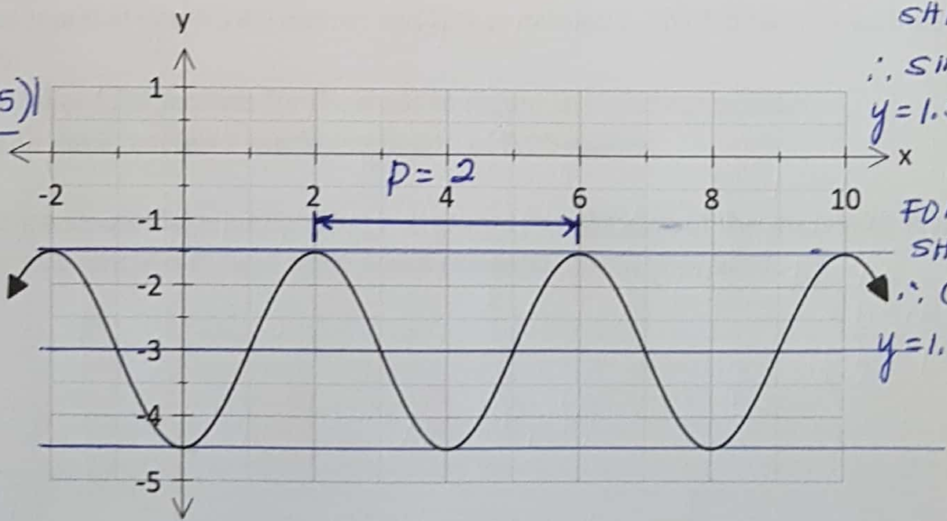
$y = 2\sin 1(x - 0) + 1$

FOR COSINE, PHASE SHIFT IS $C = \frac{\pi}{2}$.

$\therefore$ COSINE FUNCTION

$y = 2\sin 1\left(x - \frac{\pi}{2}\right) + 1$

5. Give equations of a sinusoidal function in terms of **BOTH** $\sin x$ and $\cos x$ that would match the graph given below:



$amp = \frac{|max - min|}{2}$

$= \frac{|-1.5 - (-4.5)|}{2}$

$a = 1.5$

$d = \frac{-1.5 + -4.5}{2}$

$d = -3$

Period $= 2$

$b = \frac{2\pi}{P}$

$b = \frac{2\pi}{2} = \pi$

FOR SINE, PHASE SHIFT IS 1

$\therefore$ SINE EQUATION

$y = 1.5\sin\pi(x - 1) - 3$

FOR COSINE, PHASE SHIFT IS 2

$\therefore$ COSINE EQUATION

$y = 1.5\cos\pi(x - 2) - 3$

6. Give equations of a sinusoidal function in terms of **BOTH** $\sin x$ and $\cos x$ that would match the graph given below:

Half of period $4(20) = 80$ | Full period $= 160$

$amp = \dfrac{3500 - 2500}{2}$

$a = 500$

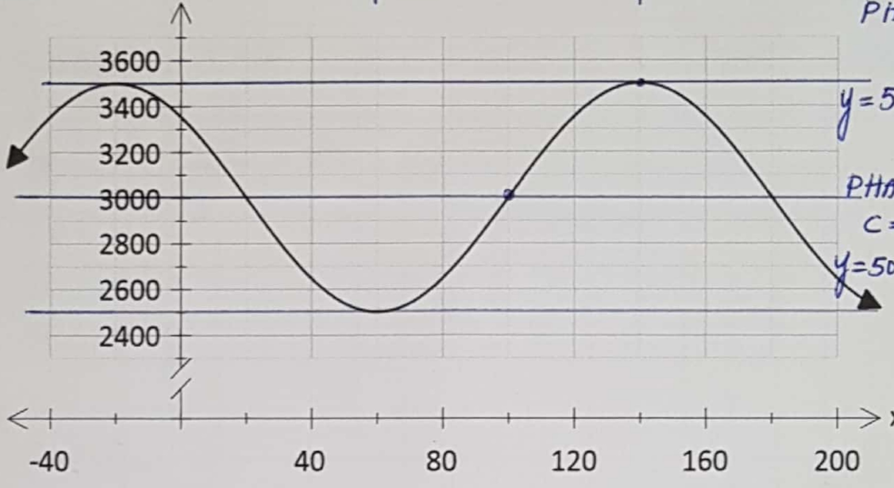
$d = \dfrac{3500 + 2500}{2}$

$d = 3000$

$P = 160$ $\quad b = \dfrac{2\pi}{P}$

$b = \dfrac{2\pi}{160}$

$b = \dfrac{\pi}{80}$



PHASE SHIFT FOR SINE

$C = 100$

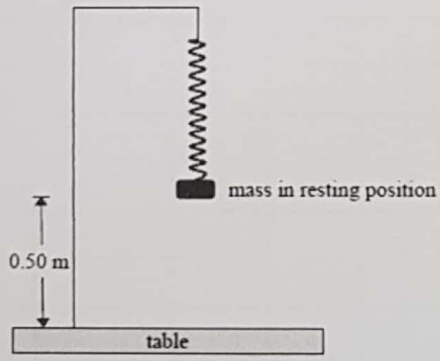
$y = 500 \sin \frac{\pi}{80}(x - 100) + 3000$

PHASE SHIFT FOR COSINE

$C = 140$

$y = 500 \cos \frac{\pi}{80}(x - 140) + 3000$

7. A mass is suspended by a spring and is in a resting position 0.50 metres above a table.



The mass is pulled down 0.40 metres and is then released. The following information is obtained:

- It takes 1.20 seconds for the mass to return to its lowest position.
- The mass reaches a maximum height of 0.90 metres.

Determine a sinusoidal equation that represents the distance of the mass with respect to the table as a function of time since it was released. Show your work.

1) Sketch the graph $y = 2\cos\frac{1}{2}(x) + 1$ from $[0, 2\pi]$

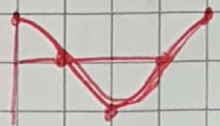
amplitude $= 2$

median line at $y = 1$

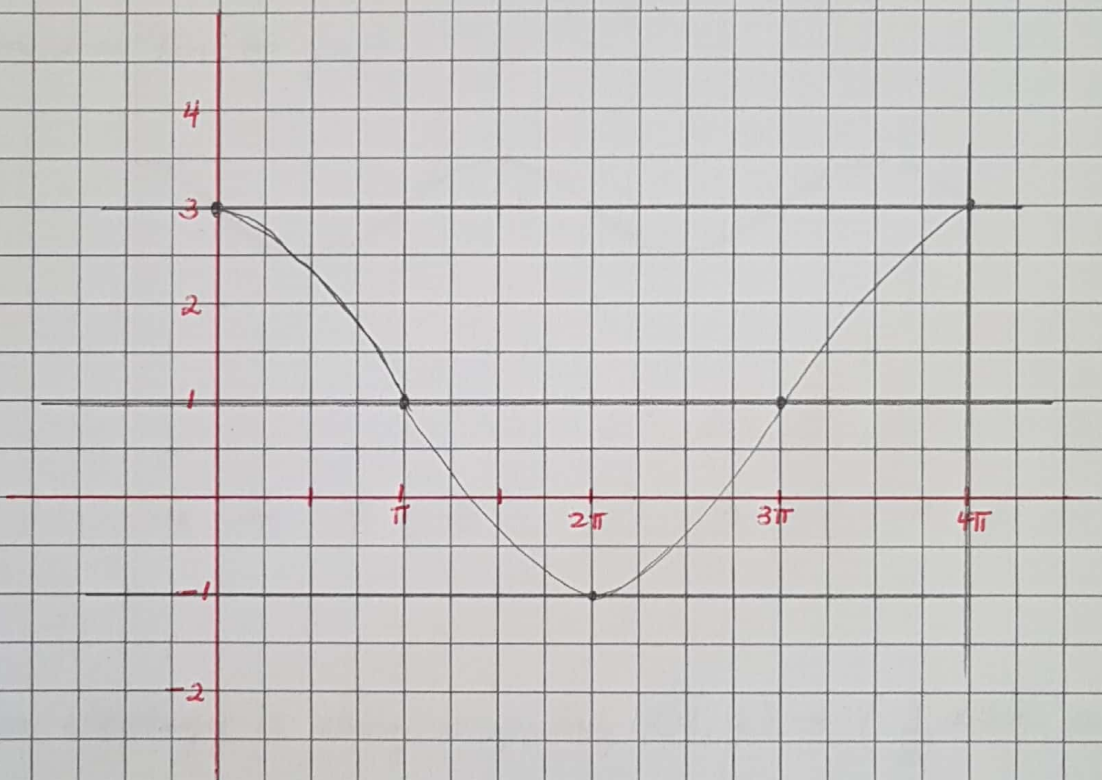
No phase shift; $x = 0$

$P = \frac{2\pi}{b} \Rightarrow P = \frac{2\pi}{\frac{1}{2}} = 4\pi$

* Remember basic cos function graph



5 key points and shape. Cosine graph starts at maximum



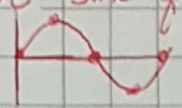
Another strategy is using mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h,\ ay + k\right)$

(x, y)	$(2x + 0,\ 2y + 1)$
$(0, 1)$	$(0, 3)$
$(\frac{\pi}{2}, 0)$	$(2(\frac{\pi}{2}), 2(0)+1) = (\pi, 1)$
$(\pi, -1)$	$(2(\pi), 2(-1)+1) = (2\pi, -1)$
$(\frac{3\pi}{2}, 0)$	$(2(\frac{3\pi}{2}), 2(0)+1) = (3\pi, 1)$
$(2\pi, 1)$	$(2(2\pi), 2(1)+1) = (4\pi, 3)$

$a = 2$
$b = 1/2$
$c = h = 0$
$d = k = 1$

2) Sketch $y = -\frac{1}{3}\sin\left(x - \frac{\pi}{4}\right) + 2$ for $[-2\pi, 2\pi]$

Basic sine function

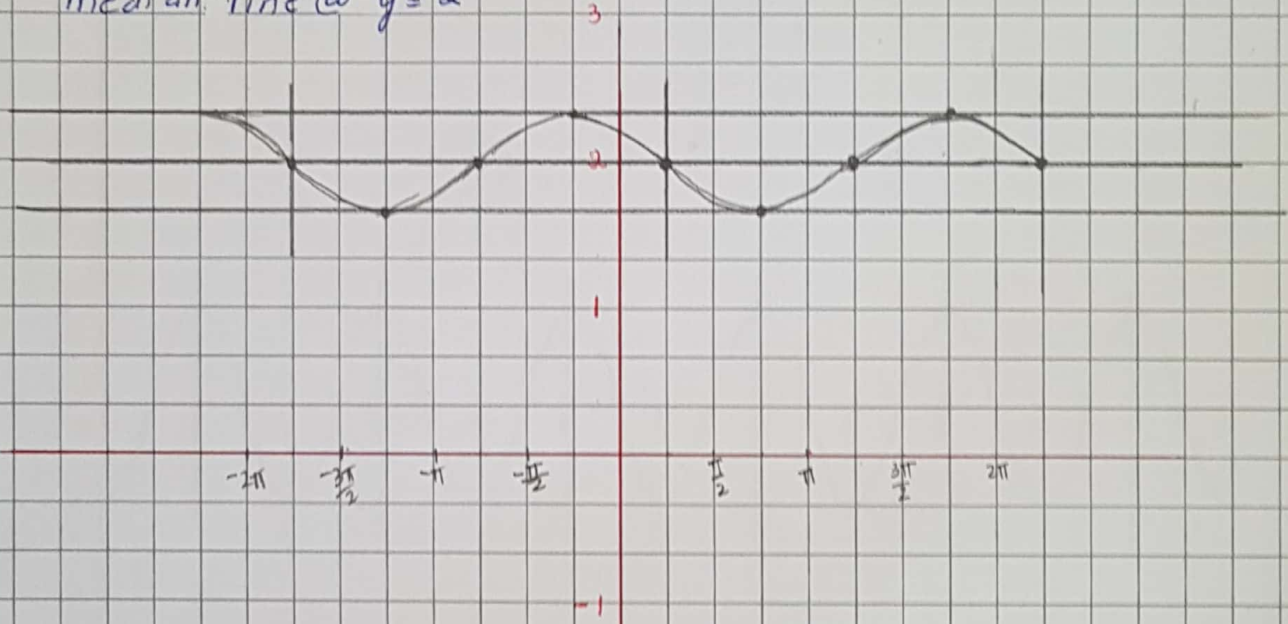


amplitude $= \frac{1}{3}$ [note: amplitude is positive].
In this case since a is negative it means we need to do a vertical reflection.

$b = 1$ $P = \frac{2\pi}{1} = 2\pi$

phase shift $= \frac{\pi}{4}$ [Note: since phase shift is $\frac{\pi}{4}$, we can use increments of $\frac{\pi}{4}$ for x-axis]

median line @ $y = 2$



Another strategy is using mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h,\ ay + k\right)$

(x, y)	$\left(x + \pi/4,\ -1/3\, y + 2\right)$
$(0, 0)$	$\left(\frac{\pi}{4}, 2\right)$
$\left(\frac{\pi}{2}, 1\right)$	$\left(\frac{\pi}{2} + \frac{\pi}{4}, -\frac{1}{3}(1) + 2\right) = \left(\frac{3\pi}{4}, 1\frac{2}{3}\right)$
$(\pi, 0)$	$\left(\pi + \frac{\pi}{4}, -\frac{1}{3}(0) + 2\right) = \left(\frac{5\pi}{4}, 2\right)$
$\left(\frac{3\pi}{2}, -1\right)$	$\left(\frac{3\pi}{2} + \frac{\pi}{4}, -\frac{1}{3}(-1) + 2\right) = \left(\frac{7\pi}{4}, 2\frac{1}{3}\right)$
$(2\pi, 0)$	$\left(2\pi + \frac{\pi}{4}, -\frac{1}{3}(0) + 2\right) = \left(\frac{9\pi}{4}, 2\right)$

$a = -\frac{1}{3}$

$b = 1$

$c = \frac{\pi}{4}$

$d = 2$

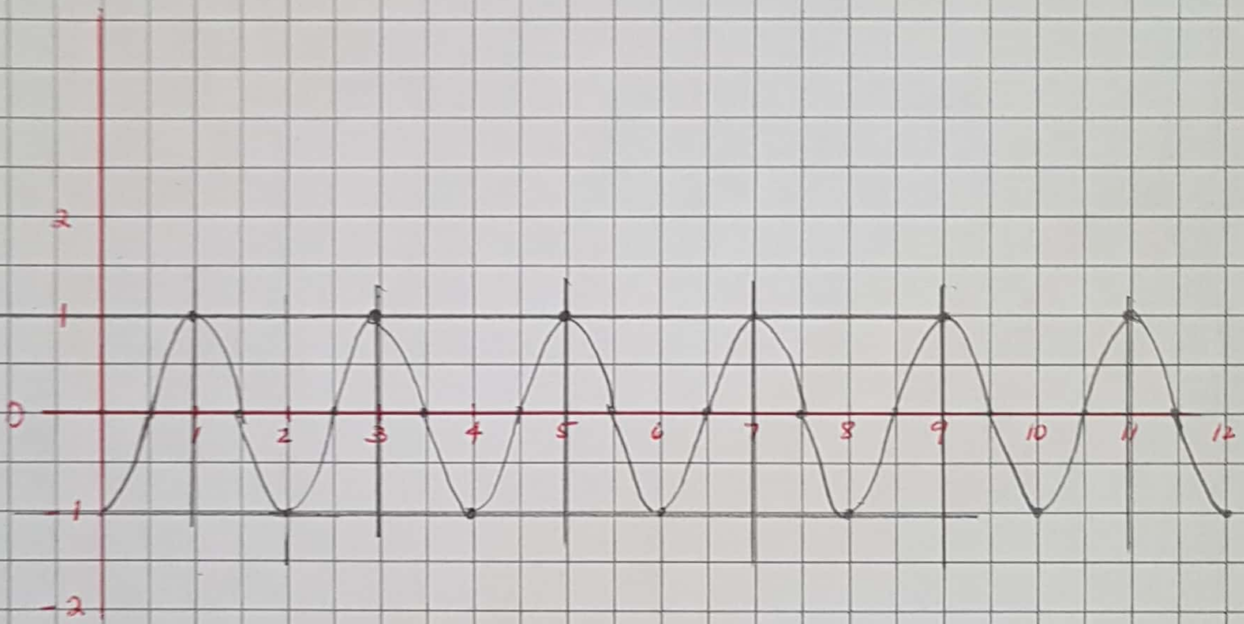
3) Sketch $y = \cos \pi (x - 3)$ from $0 \leq x \leq 4\pi$

amplitude = 1

median line: $y = 0$

$b = \pi \Rightarrow P = \frac{2\pi}{\pi} = 2$ [Note: we don't need fraction of π for the increment of x-axis]

phase shift = 3



Alternative strategy is by mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$