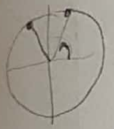
Example 5: Solving a Trigonometric Equation With a Multiple Angle

a) Solve the equation $\sin 3\theta = \frac{\sqrt{3}}{2}$ for $[0, 2\pi)$. ← Doman



Let $x = 3\theta$

$\sin x = \frac{\sqrt{3}}{2}$ $\sin x$ is +'ve in QI & II

In QI: $x = \frac{\pi}{3}$ In QII $x = \frac{2\pi}{3}$

Replace x by 3θ

$3\theta = \frac{\pi}{3}$ $3\theta = \frac{2\pi}{3}$

$\theta = \frac{\pi}{9}$ $\theta = \frac{2\pi}{9}$

Check: $\frac{\pi}{9}$

LS	RS
$\sin 3\left(\frac{\pi}{9}\right)$	$\frac{\sqrt{3}}{2}$
$\sin \frac{\pi}{3}$	
$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$

LS = RS ✓

Check $\frac{2\pi}{9}$ VERIFY.

LS	RS
$\sin 3\left(\frac{2\pi}{9}\right)$	$\frac{\sqrt{3}}{2}$
$\sin \frac{2\pi}{3}$	
$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$ ✓

If we need the general solution: $\frac{\pi}{9} + \frac{2\pi}{3}k,\ k \in \mathbb{Z}$

$P = \frac{2\pi}{b} = \frac{2\pi}{3}$ $\frac{2\pi}{9} + \frac{2\pi}{3}k,\ k \in \mathbb{Z}$

b) Solve the equation $3\cos\frac{1}{2}x - \sqrt{2} = 5\cos\frac{1}{2}x$ for $[0°, 360°)$

Let $\theta = \frac{1}{2}x$

$3\cos\theta - \sqrt{2} = 5\cos\theta$

$3\cos\theta - 5\cos\theta = \sqrt{2}$

$-2\cos\theta = \sqrt{2}$

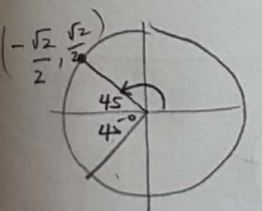
$\cos\theta = -\frac{\sqrt{2}}{2}$

$\cos\theta$ is -'ve in QII & QIII

Let's find Ref. angle.

$\cos\theta_R = \frac{\sqrt{2}}{2}$

$\theta_R = 45°$



In QII:

$\theta = 180 - 45$

$\theta = 135°$

$\frac{1}{2}x = 135°$

$x = 270°$

In QIII

$\theta = 180 + 45$

$\theta = 225°$

$\frac{1}{2}x = 225°$

$x = 450°$ ← Not part of the domain

VERIFY: $x = 270°$

LS	RS
$3\cos\frac{1}{2}(270) - \sqrt{2}$	$5\cos\frac{1}{2}(270)$
$3\cos 135° - \sqrt{2}$	$5\cos 135$
$3\left(-\frac{\sqrt{2}}{2}\right) - \sqrt{2}$	$5\left(-\frac{\sqrt{2}}{2}\right)$
$-\frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{1}$	$-\frac{5\sqrt{2}}{2}$
$-\frac{3\sqrt{2}}{2} - \frac{2\sqrt{2}}{2}$	
$-\frac{5\sqrt{2}}{2}$	
LS	RS ✓

What is the general solution?

$P = \frac{2\pi}{b}$

$P = \frac{360°}{\frac{1}{2}}$

$P = 720°$

$x = 270° + 720°k,\ k \in \mathbb{Z}$

$x = 450 + 720k,\ k \in \mathbb{Z}$

quadratic trig equation.

Example 6: Solving a Second-Degree Trigonometric Equation

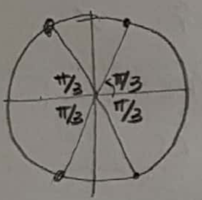
a) Solve the equation $4\sin^2 x = 3$ for $0 \le x < 2\pi$, then give the general solution.

$4\sin^2 x = 3$

$\sin^2 x = \frac{3}{4}$

$\sqrt{(\sin x)^2} = \sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$



$\sin x = \frac{\sqrt{3}}{2}$ In QI & QII

Ref angle $= \pi/3$

$\sin x = -\frac{\sqrt{3}}{2}$ In QIII & QIV

Therefore

In QI: $x = \frac{\pi}{3}$

QII: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

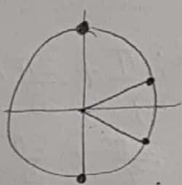
QIII: $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

QIV: $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

b) Solve the equation $2\cos^2\theta - \sqrt{3}\cos\theta = 0$ over the interval $[0°, 360°)$

$2\cos^2\theta - \sqrt{3}\cos\theta = 0$

$\cos\theta\,(\underbrace{2\cos\theta - \sqrt{3}}) = 0$



$\cos\theta = 0$

$\boxed{\theta = 90°, 270°}$

$2\cos\theta - \sqrt{3} = 0$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\boxed{\theta = 30°, 330°}$

VERIFY: $\theta = 90°$

LS	RS
$2\cos^2 90 - \sqrt{3}\cos 90$	0
$2(\cos 90)^2 - \sqrt{3}\cos 90$	
$2(0)^2 - \sqrt{3}(0)$	
0	0 ✓
$2(\cos 30)^2 - \sqrt{3}\cos 30$	
0	0

c) Solve $\sin^2\theta - \sin\theta = 2$ for $[0, 2\pi)$.

$(\sin\theta)^2 - \sin\theta = 2$

$(\sin\theta)^2 - \sin\theta - 2 = 0$

Rewrite: Let $x = \sin\theta$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

Put the $\sin\theta$ back

$(\sin\theta - 2)(\sin\theta + 1) = 0$

$\sin\theta - 2 = 0$

$\sin\theta = 2$

$\theta =$ No solution

$\sin\theta + 1 = 0$

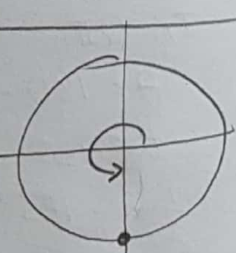
$\sin\theta = -1$

$\theta = \frac{3\pi}{2}$

VERIFY:

LS	RS
$\left(\sin\frac{3\pi}{2}\right)^2 - \sin\frac{3\pi}{2}$	2
$(-1)^2 - (-1)$	
$1 + 1$	
2	

$LS = RS$



Example 7: Solving a Second-Degree Trigonometric Equation Using the Quadratic Formula

Solve the equation $4\tan^2 x = 2\tan x + 1$ over the domain $\boxed{0 \le x < 2\pi}$, then state the general solution.

$$4(\tan x)^2 = 2\tan x + 1$$

$$4(\tan x)^2 - 2\tan x - 1 = 0$$

Let $y = \tan x$

$$4y^2 - 2y - 1 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-1)}}{2(4)}$$

$$y = \frac{2 \pm \sqrt{20}}{8}$$

$$y = \frac{2 \pm 2\sqrt{5}}{8}$$

$$y = \frac{2(1 \pm \sqrt{5})}{8}$$

$$y = \frac{1 \pm \sqrt{5}}{4}$$

Replace the $\tan x$ back

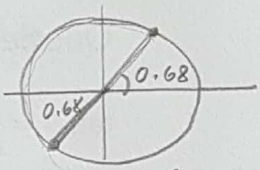
$$\tan x = \frac{1+\sqrt{5}}{4}$$

$\boxed{\text{In QI \& III}}$

$$\tan x = 0.8091$$

$$x = \tan^{-1}(0.8091)$$

$$x = 0.68$$

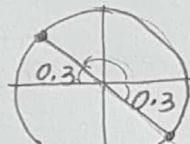


The general solution

$$x = 0.68 + \pi k, \ k \in \mathbb{Z}$$

Period for tan is π.

$$\tan x = \frac{1-\sqrt{5}}{4}$$



$$\tan x = -0.3090\ldots$$

$$x = \tan^{-1}(-0.3090)$$

$$x_R = \tan^{-1}(0.3090)$$

$$x = \pi - 0.3 = 2.84$$

The general solution.

$$x = 2.84 + \pi k, \ k \in \mathbb{Z}$$

period for tan is π.

Assignment Time!: Work on p.592- 4-15, MC 1&2

Q 9-14 on pg. 595