Lesson 2: Introduction to Trigonometric Identities

A trigonometric identity is a trigonometric equation that is true for all permissible values of the variable in the expressions on both sides of the equation. We use trigonometric identities to simplify trigonometric expressions and to solve trigonometric equations.

We have already worked with some of the simpler identities.

Reciprocal Identities

$\sin\theta = \dfrac{1}{\csc\theta}$ $\quad\cos\theta = \dfrac{1}{\sec\theta}$ $\quad\tan\theta = \dfrac{1}{\cot\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\quad\sec\theta = \dfrac{1}{\cos\theta}$ $\quad\cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\quad\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

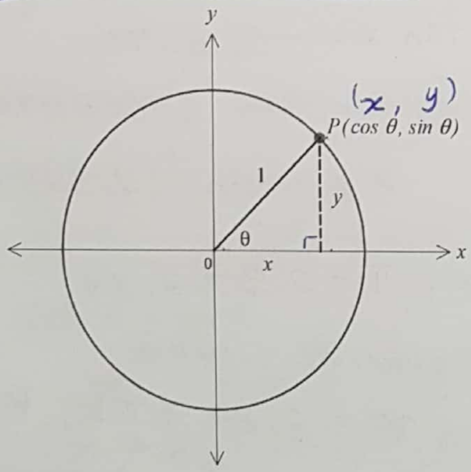
The reciprocal and quotient identities can be arranged and written in other forms:

$\csc\theta\sin\theta = 1$ $\quad \dfrac{1}{\cancel{\sin\theta}}\cancel{\sin\theta} = 1$

$\sec\theta\cos\theta = 1$ $\quad \dfrac{1}{\cancel{\cos\theta}}\cancel{\cos\theta} = 1$

$\cot\theta\tan\theta = 1$ $\quad \dfrac{\cancel{\cos\theta}}{\cancel{\sin\theta}}\dfrac{\cancel{\sin\theta}}{\cancel{\cos\theta}} = 1$

The Pythagorean Identity



Recall that point P on the terminal arm of an angle θ in standard position has coordinates $(\cos\theta, \sin\theta)$. The hypotenuse is 1 because it's the radius of the unit circle.

Using the Pythagorean Theorem:

$$x^2 + y^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1$$

This is called the Pythagorean Identity.

There are three forms of the Pythagorean Identity:

$\cos^2\theta + \sin^2\theta = 1$ $\quad \cot^2\theta + 1 = \csc^2\theta$ $\quad 1 + \tan^2\theta = \sec^2\theta$

(Note that these identities can also be re-arranged.)

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$\cot^2\theta - \csc^2\theta = -1$

$\cot^2\theta = \csc^2\theta - 1$

$\csc^2\theta - \cot^2\theta = 1$

$\tan^2\theta = \sec^2\theta - 1$

$\sec^2\theta - \tan^2\theta = 1$

$\tan^2\theta - \sec^2\theta = -1$

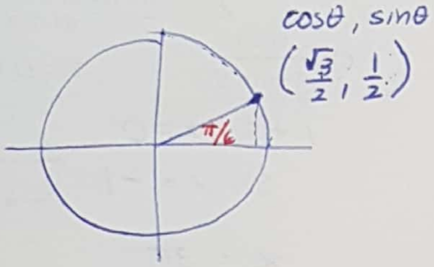
means to check.

Lesson 3: Verifying Trigonometric Identities for a Given Value

To verify means to show that an equation is true for a specific value of the variable. It does NOT prove that the equation is true for all values (i.e. does not prove that the equation is an identity).

Example 1:

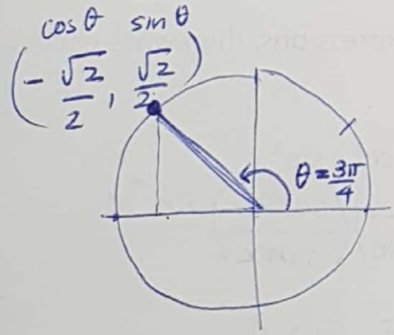
Verify that the equation $\cot^2\theta + 1 = \csc^2\theta$ is true when $\theta = \frac{\pi}{6}$.



LS	RS
$\cot^2\theta + 1$	$\csc^2\theta$
$\cot^2\left(\frac{\pi}{6}\right) + 1$	$\csc^2\left(\frac{\pi}{6}\right)$
$(\sqrt{3})^2 + 1$	$(2)^2$
4 =	4 ✓

Example 2:

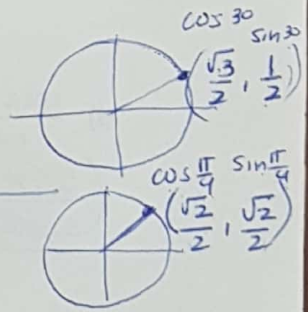
Verify that the equation $\tan^2\theta + 1 = \sec^2\theta$ is true when $\theta = \frac{3\pi}{4}$.



LS	RS
$\tan^2\theta + 1$	$\sec^2\theta$
$\tan^2\frac{3\pi}{4} + 1$	$\sec^2\frac{3\pi}{4}$
$(-1)^2 + 1$	$\left(-\frac{2}{\sqrt{2}}\right)^2$
2	$\frac{4}{2}$ ✓
	2

Example 3:

Verify that the equation $\frac{\sec x}{\tan x + \cot x} = \sin x$ is true for $x = 30°$ and for $= \frac{\pi}{4}$.



LS	RS
$\frac{\sec 30}{\tan 30 + \cot 30}$	$\sin 30$
$\dfrac{\frac{2}{\sqrt{3}}}{\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}\cdot\frac{\sqrt{3}}{\sqrt{3}}}$	$\frac{1}{2}$
$\dfrac{\frac{2}{\sqrt{3}}}{\frac{4}{\sqrt{3}}} = \frac{1}{2}$	

LS	RS
$\dfrac{\sec\frac{\pi}{4}}{\tan\frac{\pi}{4} + \cot\frac{\pi}{4}}$	$\sin\frac{\pi}{4}$
$\dfrac{\frac{2}{\sqrt{2}}}{1 + 1}$	$\frac{\sqrt{2}}{2}$
$\dfrac{\frac{2}{\sqrt{2}}}{2} = \frac{\cancel{2}}{\sqrt{2}} \times \frac{1}{\cancel{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	

Lesson 4: Simplifying Trigonometric Expressions

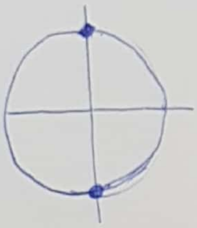
Many trigonometric expressions involve fractions. Remember that the denominator of a fraction cannot equal zero. Also, the ratios $\tan\theta$, $\cot\theta$, $\sec\theta$, *and* $\csc\theta$ all have values of θ that cause their value to be *undefined* – those values of θ are also non-permissible values.

Example 1

Determine the non-permissible values of x for each expression: the denominator is zero. Always look for fraction and its denominator

a) $\dfrac{\sin x}{\cos x}$

what values of x that will make $\cos x$ equal to zero?



$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

gen. $x = \frac{\pi}{2} + \pi k,\ k \in \mathbb{Z}$

b) $\dfrac{\cos x}{\tan x}$

npv

$\tan x \neq 0$

$\dfrac{\sin x}{\cos x}$

$\sin x = 0$

$0, \pi, 2\pi, \dots$

$\cos x = 0$

$\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

c) $\dfrac{\cot x}{1+\sin x}$

$\cot x = \dfrac{\cos x}{\sin x}$

$\sin x = 0$

$x = 0, \pi, 2\pi, \dots$

$1 + \sin x = 0$

$\sin x = -1$

$x = \frac{3\pi}{2},$

general.

$\frac{3\pi}{2} + 2\pi k,\ k \in \mathbb{Z}$

Example 2

Determine any non-permissible values of x in each of the following expressions, then write each expression in terms of a single trigonometric function.

a) $\cot x \sin x$

$= \dfrac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}$

$= \cos x$

b) $\dfrac{\cos x}{\cot x}$

$= \dfrac{\cos x}{\frac{\cos x}{\sin x}}$

$= \cancel{\cos x}\left(\dfrac{\sin x}{\cancel{\cos x}}\right) = \sin x$

c) $\sec x \cot x \sin^2 x$

$\left(\dfrac{1}{\cancel{\cos x}}\right)\left(\dfrac{\cancel{\cos x}}{\cancel{\sin x}}\right)\sin^{\cancel{2}} x$

$= \sin x.$

d) $\dfrac{\cot x \tan x}{\csc x}$

$\dfrac{\left(\frac{\cancel{\cos x}}{\cancel{\sin x}}\right)\left(\frac{\cancel{\sin x}}{\cancel{\cos x}}\right)}{\frac{1}{\sin x}} = \dfrac{1}{\frac{1}{\sin x}}$

$= \sin x$

e) $\dfrac{\sec^2 x \cos x}{\csc x}$

$\dfrac{\frac{1}{\cos^{\cancel{2}} x}(\cancel{\cos x})}{\frac{1}{\sin x}} = \dfrac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \dfrac{1}{\cos x}\left(\dfrac{\sin x}{1}\right)$

$= \tan x$

f) $(\csc x)(\cot x)(\sec x)(\sin x)$

$= \left(\dfrac{1}{\sin x}\right)\left(\dfrac{\cancel{\cos x}}{\cancel{\sin x}}\right)\left(\dfrac{1}{\cancel{\cos x}}\right)(\cancel{\sin x})$

$= \csc x$

$$\boxed{\begin{array}{l}\text{Practice} \\ \text{Pg } 611-613 \\ \text{Q \# } 3, 4, 5\end{array}}$$