Lesson 5: Proving Trigonometric Identities

Example 1:

Prove the following identities. Identify any non-permissible values.

* start w/ more complicated side

* NO transfering terms on either side of equation

a) $\underbrace{\cos x \sec x}_{LS} = \underbrace{1}_{RS}$

LS	RS
$\cos x \sec x$	1
$= \cancel{\cos x}\left(\dfrac{1}{\cancel{\cos x}}\right)$	
$= 1$	1

$LS = RS$

b) $\underbrace{\tan x \cos x}_{LS} = \underbrace{\sin x}_{RS}$

LS	RS
$\tan x \cos x$	$\sin x$
$= \dfrac{\sin x}{\cancel{\cos x}} \cancel{\cos x}$	
$= \sin x$	$\sin x$

$LS = RS$

c) $\underbrace{\dfrac{\sec x}{\csc x}}_{LS} = \underbrace{\tan x}_{RS}$

LS	RS
$\dfrac{\sec x}{\csc x}$	$\tan x$
$= \dfrac{\frac{1}{\cos x}}{\frac{1}{\sin x}}$	
$= \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{1}$	
$= \dfrac{\sin x}{\cos x}$	
$= \tan x$	$\tan x$

$LS = RS$

d) $\underbrace{\cos^2 x}_{LS} = \underbrace{\dfrac{\cot x \sin x}{\sec x}}_{RS}$ ← break it down to just $\sin x$ & $\cos x$

LS	RS
$\cos^2 x$	$\dfrac{\frac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x}}{\frac{1}{\cos x}}$
	$= \dfrac{\cos x}{\frac{1}{\cos x}}$
	$= \cos x \cdot \dfrac{\cos x}{1}$
$\cos^2 x$	$= \cos^2 x$
LS	RS

e) $\underbrace{\csc x \cos^2 x + \sin x}_{} = \underbrace{\csc x}_{}$

RS	LS
$\csc x \cos^2 x + \sin x$	
$= \frac{1}{\sin x} \cos^2 x + \sin x$	
$= \frac{\cos^2 x}{\sin x} + \frac{\sin x}{1}$	
$= \frac{\cos^2 x}{\sin x} + \frac{\sin x}{1}\left(\frac{\sin x}{\sin x}\right)$	
$= \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x}$	
$= \frac{\cos^2 x + \sin^2 x}{\sin x}$	
$= \frac{1}{\sin x}$	
$= \csc x$	$\csc x$

LS = RS

f) $\underbrace{\cot^3\theta}_{} = \underbrace{\cot\theta \csc^2\theta - \cot\theta}_{}$

LS	RS
$\cot^3\theta$	$\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin^2\theta} - \frac{\cos\theta}{\sin\theta}$
	$= \frac{\cos\theta}{\sin^3\theta} - \frac{\cos\theta}{\sin\theta}\left(\frac{\sin^2\theta}{\sin^2\theta}\right)$
	$= \frac{\cos\theta}{\sin^3\theta} - \frac{\cos\theta \sin^2\theta}{\sin^3\theta}$
	$= \frac{\cos\theta - \cos\theta\sin^2\theta}{\sin^3\theta}$
	$= \frac{\cos\theta(1-\sin^2\theta)}{\sin^3\theta}$
	$= \frac{\cos\theta(\cos^2\theta)}{\sin^3\theta}$
	$= \frac{\cos^3\theta}{\sin^3\theta}$
$\cot^3\theta$	$= \cot^3\theta$
LS	= RS

There is another way of proving this identity. See separate page.

g) $\dfrac{\cos x \sin x}{1+\sin x} = \dfrac{1-\sin x}{\cot x}$

LS	RS
$= \dfrac{\cos x \sin x}{(1+\sin x)} \cdot \dfrac{\cos x}{\cos x}$	
$= \dfrac{\cos^2 x \sin x}{(1+\sin x)\cos x}$	
$= \dfrac{(1-\sin^2 x)(\sin x)}{(1+\sin x)\cos x}$	
$= \dfrac{\cancel{(1+\sin x)}(1-\sin x)(\sin x)}{\cancel{(1+\sin x)}(\cos x)}$	
$= (1-\sin x)\left(\dfrac{\sin x}{\cos x}\right)$	
$= (1-\sin x)(\tan x)$	
$= (1-\sin x)\left(\dfrac{1}{\cot x}\right)$	
$= \dfrac{1-\sin x}{\cot x}$	$\dfrac{1-\sin x}{\cot x}$

LS = RS

h) $\sin x + \dfrac{\cot x}{\sec x} = \csc x$

LS	RS
$\sin x + \dfrac{\cot x}{\sec x}$	$\csc x$
$= \sin x + \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\cos x}}$	
$= \sin x + \dfrac{\cos x}{\sin x} \cdot \dfrac{\cos x}{1}$	
$= \dfrac{\sin x}{1} + \dfrac{\cos^2 x}{\sin x}$	
$= \dfrac{\sin x}{1}\left(\dfrac{\sin x}{\sin x}\right) + \dfrac{\cos^2 x}{\sin x}$	
$= \dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x}$	
$= \dfrac{\sin^2 x + \cos^2 x}{\sin x}$	
$= \dfrac{1}{\sin x}$	
$= \csc x$	$\csc x$

LS = RS

Pls. find another solution on our Notes.

Assignment Time! Work on: p.611- 3-9, 11, MC1&2;
p.626- 3, 4a) and c), 5, 6, 7, 9, 12a, MC1&2

Pg 13) Alternate solution

f) $\cot^3\theta = \cot\theta\csc^2\theta - \cot\theta$

LS	RS
	$\cot\theta \cdot \frac{1}{\sin^2\theta} - \cot\theta$
	$= \frac{\cot\theta}{\sin^2\theta} - \frac{\cot\theta}{1}\left(\frac{\sin^2\theta}{\sin^2\theta}\right)$
	$= \frac{\cot\theta}{\sin^2\theta} - \frac{\cot\theta\sin^2\theta}{\sin^2\theta}$
	$= \frac{\cot\theta - \cot\theta\sin^2\theta}{\sin^2\theta}$
	$= \frac{\cot\theta(1-\sin^2\theta)}{\sin^2\theta}$
	$= \frac{\cot\theta(\cos^2\theta)}{\sin^2\theta}$
	$= \cot\theta(\cot^2\theta)$
$\cot^3\theta$	$= \cot^3\theta$

LS = RS

pg 14) Alternate solution

g) $$\frac{\cos x \sin x}{1+\sin x} = \frac{1-\sin x}{\cot x}$$

$$= \frac{1-\sin x}{\frac{\cos x}{\sin x}}$$

$$= (1-\sin x)\left(\frac{\sin x}{\cos x}\right)$$

$$= (1-\sin x)\left(\frac{\sin x}{\cos x}\right)\left(\frac{\cos x}{\cos x}\right)$$

$$= \frac{(1-\sin x)(\sin x)(\cos x)}{\cos^2 x}$$

$$= \frac{(1-\sin x)(\sin x)(\cos x)}{1-\sin^2 x} \leftarrow$$ factor diff of squares

$$= \frac{\cancel{(1-\sin x)}(\sin x)(\cos x)}{\cancel{(1-\sin x)}(1+\sin x)}$$ cancel common factors

$$= \frac{\sin x \cos x}{1+\sin x}$$

LS = RS