

Lesson 6: Using Identities to Solve Equations

Example 1: Solve each equation over the domain $0 \leq x \leq 2\pi$.

a) $3 \cos x + 1 = 2 \sec x$

We know $\sec x = \frac{1}{\cos x}$

$$3 \cos x + 1 = 2 \left(\frac{1}{\cos x} \right)$$

$$\cos x (3 \cos x + 1) = \left(\frac{2}{\cos x} \right) \cos x$$

$$3 \cos^2 x + \cos x = 2$$

$$3 \cos^2 x + \cos x - 2 = 0$$

$$(3 \cos x - 2)(\cos x + 1) = 0$$

$$\cos x = \frac{2}{3}$$

$$\cos x = -1$$

b) $2 \cos^2 x - 3 \sin x = 0$

We know $\cos^2 x = 1 - \sin^2 x$

$$2(1 - \sin^2 x) - 3 \sin x = 0$$

$$2 - 2 \sin^2 x - 3 \sin x = 0$$

$$-2 \sin^2 x - 3 \sin x + 2 = 0$$

$$2 \sin^2 x + 3 \sin x - 2 = 0$$

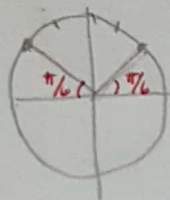
$$(2 \sin x - 1)(\sin x + 2) = 0$$

$$2 \sin x - 1 = 0$$

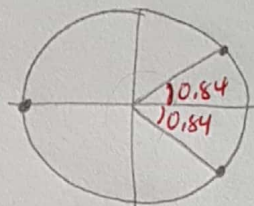
$$\sin x = \frac{1}{2}$$

$$\sin x + 2 = 0$$

$$\sin x = \frac{-2}{1}$$

 $x = \text{NO SOLUTION}$


$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

There are 2 solutions: $x = \frac{\pi}{6}, \frac{5\pi}{6}$ VISUALIZE!

$$\cos x = \frac{2}{3}$$

$$x = \cos^{-1}\left(\frac{2}{3}\right)$$

$$x_R = 0.84^\circ$$

$$\text{In QI: } x = 0.84^\circ$$

$$\text{In QIV: } x = 2\pi - 0.84 = 5.44^\circ$$

There are 3 solutions:

$$x = \pi, 0.84^\circ, 5.44^\circ$$

We know $\csc x = \frac{1}{\sin x}$

c) $2 \sin x = 3 + 2 \csc x$

$$2 \sin x = 3 + 2 \left(\frac{1}{\sin x} \right)$$

$$\sin x (2 \sin x) = \left(3 + \frac{2}{\sin x} \right) \sin x$$

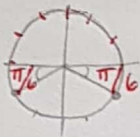
$$2 \sin^2 x = 3 \sin x + 2$$

$$2 \sin^2 x - 3 \sin x - 2 = 0$$

$$(2 \sin x + 1)(\sin x - 2) = 0$$

$$2 \sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$



Let's find the reference angle

$$\sin x_R = \frac{1}{2}$$

$$x_R = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x_R = \frac{\pi}{6}$$

Therefore

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

d) $\sin x + \sqrt{3} \cos x = 0$

$$\sin x + \sqrt{3} \cos x = 0$$

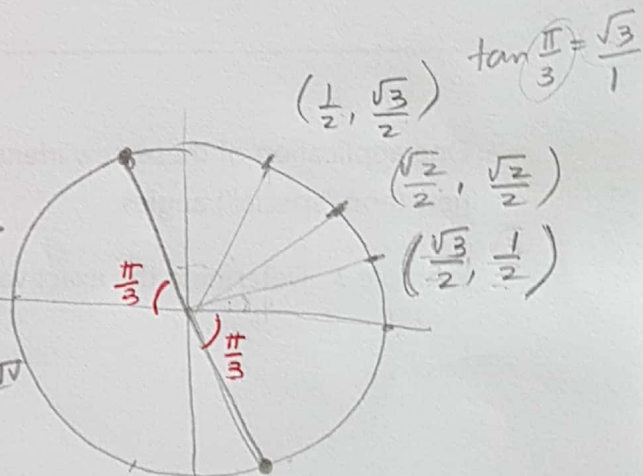
$$\frac{\sin x}{\cos x} = \frac{-\sqrt{3} \cos x}{\cos x}$$

$$\tan x = -\sqrt{3}$$

In QII: $x = \frac{2\pi}{3}$,

In QIV: $x = 2\pi - \frac{\pi}{3}$
 $= \frac{5\pi}{3}$

Where is
 $\tan x$
 NEGATIVE?
 In QII & QIV



Two solutions for $x = \frac{2\pi}{3}, \frac{5\pi}{3}$.

Assignment Time! Work on: (p.616- 10, 12 p.630- 10)