9. For each equation, determine the general solution over the set of real numbers, then list the roots over the domain $-\pi \leq x < 0$.

a) $\cos 3x - 1 = 5 \cos 3x + 2$

$\cos 3x - 5\cos 3x = 3$

$-4\cos 3x = 3$

$\cos 3x = \frac{3}{-4}$

Let $\theta = 3x$

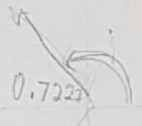
cos is -'ve in QII & QIII

$\cos\theta = \frac{3}{-4}$

Let's figure out the ref. angle first

$\cos\theta = \frac{3}{4}$

$\theta_R = 0.7227$



$\theta = \pi - 0.7227$

$= 2.4189$

$\frac{3x}{3} = \frac{2.4189}{3}$

Period $\frac{2\pi}{3}$

GEN. sol'n $x = 0.8062 + \frac{2\pi}{3}(k)$

$\theta = \pi + 0.7227$

$= 3.8643$

$3x = 3.8643$

GEN. Solution $x = 1.2881 + \frac{2\pi}{3}(k)$

b) $3 \sin 4x = 3 - 2 \sin 4x$

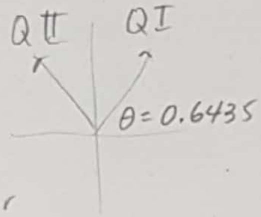
$3\sin 4x + 2\sin 4x = 3$

$5\sin 4x = 3$

$\sin 4x = \frac{3}{5}$

Let $\theta = 4x$

$\sin\theta = \frac{3}{5}$



$\theta = \sin^{-1}\left(\frac{3}{5}\right)$

$\theta = 0.6435^r$

for QI: $\theta = 0.6435$

$4x = 0.6435$

$x = 0.1609^r$

for QII:

$\theta = \pi - 0.6435$

$\theta = 2.4980$

$4x = 2.4980$

$x = 0.6245^r$

Period $= \frac{2\pi}{4} = \frac{\pi}{2}$

general solution

$x = 0.1609 + \frac{\pi}{2}(k),\ k \in \mathbb{Z}$

or

$x = 0.6245 + \frac{\pi}{2}(k),\ k \in \mathbb{Z}$.

10. Two students determined the general solution of the equation $3 \sin x + 5 = 5(\sin x + 1)$. Joseph said the solution is $x = 2\pi k$ or $x = \pi + 2\pi k$, where k is an integer. Yeoun Sun said the solution is $x = \pi k$, where k is an integer. Who is correct? Explain.

$3\sin x + 5 = 5\sin x + 5$

$3\sin x - 5\sin x = 0$

$-2\sin x = 0$

$\sin x = 0$

$x = 0^r, \pi, 2\pi, 3\pi, \ldots$

$x = \pi k,\ k \in \mathbb{Z}$.

11. Solve each equation over the domain $-\pi \leq x \leq \frac{\pi}{2}$. $[-\pi, 0] \cup [0, \frac{\pi}{2}]$

a) $4\cos^2 x - 3 = 0$

$4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

For Reference angle

$\cos x = \frac{\sqrt{3}}{2}$

$x_R = \frac{\pi}{6}$

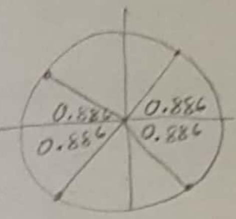
Therefore

$x = -\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{\pi}{6}$

b) $2\tan^2 x = 3$

$\tan^2 x = \frac{3}{2}$

$\tan x = \pm\sqrt{\frac{3}{2}}$



Reference angle

$x_R = \tan^{-1}\sqrt{\frac{3}{2}}$

$x_R = 0.886$

In QI: $x = 0.886$

Q III: $x = -\pi + 0.886$

$= -2.256$

Q IV: $x = -0.866$

12. Use factoring to solve each equation over the domain $-90° \leq x < 270°$.

a) $2\cos x \sin x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = -90°, 90°$ $x = 30°, 150°$

b) $3\tan x + \tan^2 x = 2\tan x$

$\tan^2 x + \tan x = 0$

$\tan x(\tan x + 1) = 0$

$\tan x = 0$ $\tan x + 1 = 0$

$\tan x = -1$

$x = 0°, 180°$ $x = +135°, \cancel{315}°$ outside the domain.

13. A student wrote the solution below to solve the equation $2\sin^2 x + \sin x = 1$ over the domain $0 \leq x < 2\pi$. Identify any errors, then write a correct solution.

$2\sin^2 x + \sin x = 1$ ← One side of Eqn should be zero

$(\sin x)(2\sin x + 1) = 1$

$\sin x = 1$ or $2\sin x + 1 = 1$

$x = \frac{\pi}{2}$ $\sin x = 0$

$x = 0$ or $x = \pi$

$2\sin^2 x + \sin x - 1 = 0$

$2(\sin x)^2 + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2}$ $\sin x = -1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x = -1$

$x = \frac{3\pi}{2}$

14. Solve each equation over the domain $-2\pi \le x \le 2\pi$, then determine the general solution.

a) $2\cos^2 x - \cos x - 1 = 0$

$2(\cos x)^2 - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$

$\cos x = \frac{-1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$-\frac{2\pi}{3}, -\frac{4\pi}{3}$

$\cos x - 1 = 0$

$\cos x = 1$

$x = 0, 2\pi, -2\pi$

General solution to find all solutions

$x = \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}$

$x = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}$

$x = 0 + 2\pi k, k \in \mathbb{Z}$

b) $5\sin^2 x + 3\sin x = 2$

$5(\sin x)^2 + 3\sin x - 2 = 0$

$(5\sin x - 2)(\sin x + 1) = 0$

$5\sin x - 2 = 0$

$\sin x = \frac{2}{5}$

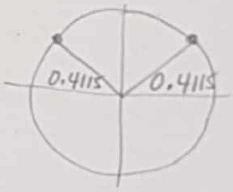
sine is +'ve in QI & QII.

Let's find Ref angle first.

$\sin x_R = \frac{2}{5}$

$x_R = \sin^{-1}\left(\frac{2}{5}\right)$

$x_R = 0.4115$



In QI: $x = 0.4115^r$

In QII: $x = \pi - 0.4115$

$= 2.730^r$

general solution to find ALL solution

$x = 0.4115 + 2\pi k, k \in \mathbb{Z}$

AND

$x = 2.730 + 2\pi k, k \in \mathbb{Z}$

$\sin x = -1$

$x = \frac{3\pi}{2}$

This is a special angle

The general solution to find all solutions

$x = \frac{3\pi}{2} + 2\pi k, k \in \mathbb{Z}$