10. Use algebra to solve each equation over the domain $0 \leq x < 2\pi$. Give the roots to the nearest hundredth where necessary.

$(\cos x - \sqrt{3}\sin x)$ multiply the conjugate

a) $\tan x = \cot x$

$\tan x = \frac{1}{\tan x}$

$\tan^2 x = 1$

$\tan^2 x - 1 = 0$

$(\tan x - 1)(\tan x + 1) = 0$

$\tan x + 1 = 0$ $\tan x - 1 = 0$

$\tan x = -1$ $\tan x = 1$

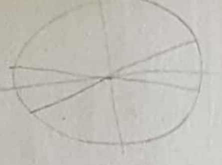
$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ $x = \frac{\pi}{4}, \frac{5\pi}{4}$

b) $\cos x + \sqrt{3}\sin x = 0$

$\cos^2 x - 3\sin^2 x = 0$

$\cos^2 x - 3(1 - \cos^2 x) = 0$

$\cos^2 x - 3 + 3\cos^2 x = 0$



$4\cos^2 x - 3 = 0$

$\sqrt{\cos^2 x} = \sqrt{\frac{3}{4}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\cos x = +\frac{\sqrt{3}}{2}$ $\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$ $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

c) $2\cos x = 7 - 3\sec x$

$2\cos x = 7 - 3\left(\frac{1}{\cos x}\right)$

$2\cos^2 x = 7\cos x - 3$

$2\cos^2 x - 7\cos x + 3 = 0$

$(2\cos x - 1)(1\cos x - 3) = 0$

$2\cos x - 1 = 0$ $\cos x = 3$

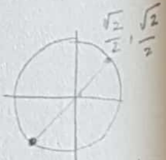
$\cos x = \frac{1}{2}$ No solution

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

d) $\sin^2 x = \sin x \cos x$

$\sin^2 x - \sin x \cos x = 0$

$\sin x(\sin x - \cos x) = 0$



$\sin x = 0$ $\sin x - \cos x = 0$

$x = 0, \pi, 2\pi$ $\sin x = \cos x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

11. Identify any errors in this proof, then write a correct algebraic proof.

To prove: $\frac{\sin\theta}{1 - \sin\theta} = \frac{1}{\csc\theta - 1}$

$$\text{L.S.} = \frac{\sin\theta}{1 - \sin\theta}$$
$$= \frac{\sin\theta}{1} - \frac{\sin\theta}{\sin\theta}$$
$$= \sin\theta - 1$$
$$= \frac{1}{\csc\theta} - 1$$
$$= \frac{1}{\csc\theta - 1}$$
$$= \text{R.S.}$$

Start w/ complicated side

$$RS = \frac{1}{\csc\theta - 1}$$
$$= \frac{1}{\frac{1}{\sin\theta} - 1\left(\frac{\sin\theta}{\sin\theta}\right)}$$
$$= \frac{1}{\frac{1}{\sin\theta} - \frac{\sin\theta}{\sin\theta}}$$
$$= \frac{1}{\frac{1 - \sin\theta}{\sin\theta}}$$
$$= \frac{\sin\theta}{1 - \sin\theta} = LS$$

12. Identify which equation below is an identity. Justify your answer. Prove the identity. Solve the other equation over the domain $-\pi \leq x \leq \pi$. Give the roots to the nearest hundredth.

a) $\dfrac{2\sin^2 x + 1}{\sin x} = 2\csc^2 x - 1$ b) $\dfrac{\sin^2 x + 1}{\sin x} = \dfrac{1 + \csc^2 x}{\csc x}$

$2\csc^2 x - 1$

$= \dfrac{2}{\sin^2 x} - 1$

$= \dfrac{2}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}$

$= \dfrac{2 - \sin^2 x}{\sin^2 x}$

$= \dfrac{2}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}$

$= 2\csc^2 x - 1$

$2\csc^2 x - 1$

LS | RS

$\dfrac{\sin^2 x + 1}{\sin x}$

$\dfrac{1 + \csc^2 x}{\csc x}$

$= \dfrac{1 + \dfrac{1}{\sin^2 x}}{\dfrac{1}{\sin x}}$

$= \dfrac{\dfrac{\sin^2 x}{\sin^2 x} + \dfrac{1}{\sin^2 x}}{\dfrac{1}{\sin x}}$

$= \dfrac{\sin^2 x + 1}{\sin^2 x} \cdot \dfrac{\sin x}{1}$

$= \dfrac{\sin^2 x + 1}{\sin x}$

C

13. Here are two identities that involve the cotangent ratio:
$\cot\theta = \dfrac{1}{\tan\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

a) Show how you can derive one identity from the other.

$\cot\theta = \dfrac{1}{\tan\theta}$

$= \dfrac{1}{\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\cos\theta}{\sin\theta}$

12. Identify which equation below is an identity. Justify your answer. Prove the identity. Solve the other equation over the domain $-\pi \leq x \leq \pi$. Give the roots to the nearest hundredth.

a) $\dfrac{2\sin^2 x + 1}{\sin x} = 2\csc^2 x - 1$ b) $\dfrac{\sin^2 x + 1}{\sin x} = \dfrac{1 + \csc^2 x}{\csc x}$

$2\csc^2 x - 1$

$= \dfrac{2}{\sin^2 x} - 1$

$= \dfrac{2}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}$

$= \dfrac{2 - \sin^2 x}{\sin^2 x}$

$= \dfrac{2}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}$

$= 2\csc^2 x - 1$

LS | RS

$2\csc^2 x - 1$

$\dfrac{\sin^2 x + 1}{\sin x}$

$\dfrac{1 + \csc^2 x}{\csc x}$

$= \dfrac{1 + \dfrac{1}{\sin^2 x}}{\dfrac{1}{\sin x}}$

$= \dfrac{\dfrac{\sin^2 x}{\sin^2 x} + \dfrac{1}{\sin^2 x}}{\dfrac{1}{\sin x}}$

$= \dfrac{\sin^2 x + 1}{\sin^2 x} \cdot \dfrac{\sin x}{1}$

$= \dfrac{\sin^2 x + 1}{\sin x}$

C

13. Here are two identities that involve the cotangent ratio: $\cot\theta = \dfrac{1}{\tan\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

a) Show how you can derive one identity from the other.

$\cot\theta = \dfrac{1}{\tan\theta}$

$= \dfrac{1}{\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\cos\theta}{\sin\theta}$