

10. Use algebra to solve each equation over the domain

$-90^\circ \leq x \leq 270^\circ$. Give the roots to the nearest degree.

a) $4 - 4 \cos^2 x = \sin x$

$$4 - 4(1 - \sin^2 x) = \sin x$$

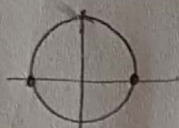
$$4 - 4 + 4\sin^2 x - \sin x = 0$$

$$4\sin^2 x - \sin x = 0$$

$$\sin x (4\sin x - 1) = 0$$

$$\sin x = 0$$

$$x = 0^\circ, 180^\circ$$



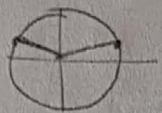
$$4\sin x - 1 = 0$$

$$\sin x = \frac{1}{4}$$

sine is positive in QI & QII

$$x_R = \sin^{-1}\left(\frac{1}{4}\right)$$

$$x_R = 14.5^\circ$$



b) $\cos x + 1 = 2 \sin^2 x$

$$\cos x + 1 = 2(1 - \cos^2 x)$$

$$\cos x + 1 = 2 - 2\cos^2 x$$

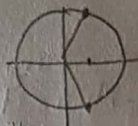
$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ, -60^\circ$$



$$\cos x + 1 = 0$$

$$\cos x = -1$$

$$x = 180^\circ$$



In QI: $x = 14.5^\circ$

In QII: $x = 180 - 14.5^\circ = 165.5^\circ$

11. Identify whether each equation is an identity. Justify your answer.

Prove each identity. Use algebra to solve each equation that is not an identity over the domain $-\pi \leq x \leq \pi$. Give the roots to the nearest hundredth.

a) $\cos^2 x = (\sin x)(\csc x + \sin x)$ PROVE

$$\begin{aligned} \cos^2 x &= \sin x \left(\frac{1}{\sin x} + \sin x \right) \\ &= 1 + \sin x (\sin^2 x) \\ &= 1 + \sin x (1 - \cos^2 x) \\ &= 1 + \sin x - \sin x \cos^2 x \\ &= 1 + \sin x - \sin x (1 - \sin^2 x) \\ &= 1 + \sin^3 x \end{aligned}$$

NOT AN IDENTITY

SOLVE

$$\cos x = \sin x \left(\frac{1}{\sin x} + \sin x \right)$$

$$\cos x = 1 + \sin^2 x$$

$$\cos x = 1 + (1 - \cos^2 x)$$

$$\cos x = 2 - \cos^2 x$$

$$\cos^2 x + \cos x - 2 = 0$$

$$(\cos x - 1)(\cos x + 2) = 0$$

$$\cos x = 1$$

$$\cos x = -2$$

NO SOLUTION

$$x = 0, \pi$$

but 0 and π are non-permissible values. Therefore, NO SOLUTION

b) $(\cos x)(\sec x - \cos x) = \cos^2 x$

$$\cos x \left(\frac{1}{\cos x} - \cos x \right)$$

$$1 - \cos^2 x$$

$$\sin^2 x$$

$$\cos^2 x$$

$$\cos^2 x$$

NOT AN IDENTITY

$$\left(\cos x \right) \left(\frac{1}{\cos x} - \cos x \right) = \cos^2 x$$

$$1 - \cos^2 x = \cos^2 x$$

$$0 = 2\cos^2 x - 1$$

$$\frac{1}{2} = \frac{2\cos^2 x}{2}$$

$$\pm \sqrt{\frac{1}{2}} = \sqrt{\cos^2 x}$$

$$\pm \frac{1}{\sqrt{2}} = \cos x$$

$$x = \frac{\pi}{4}, -\frac{\pi}{4}, \frac{3\pi}{4}, -\frac{3\pi}{4}$$

