For $\cos x = 0, x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$

Verify by substitution that neither value of $x$ is a root of the given equation.

The roots are: $x = \frac{2\pi}{3}$ and $x = \frac{5\pi}{3}$

Verify the roots by substitution.

Here are some strategies to use to prove an identity:

- Start by simplifying the side of the identity that is more complex.
- Write the expressions in terms of $\sin x$ and $\cos x$.

## Discuss the Ideas

1. What is the difference between a trigonometric identity and a trigonometric equation? Suppose you are given a trigonometric equation. What strategy can you use to check whether it might be an identity?

2. Can you conclude that an equation is an identity when it is shown to be valid for a given value of the variable? Explain.

## Exercises

**A**

3. Write each expression in terms of a single trigonometric function.

a) $\frac{\cos \theta}{\sin \theta}$

$= \cot \theta$

b) $\frac{\sin^2 \theta}{\cos^2 \theta} = \left(\frac{\sin \theta}{\cos \theta}\right)^2$

$= \tan^2 \theta$

c) $\sin^2\theta \sec\theta \cos\theta \csc\theta$

$\sin^2\theta \left(\frac{1}{\cos\theta}\right) \cos\theta \left(\frac{1}{\sin\theta}\right)$

$= \sin\theta$

d) $\dfrac{\sin^2\theta}{\tan^2\theta} = \dfrac{\sin^2\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$

$= \left(\frac{\sin^2\theta}{1}\right)\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$

$= \cos^2\theta$

**4.** Determine the non-permissible values of $\theta$.

a) $\sec\theta = \dfrac{1}{\cos\theta}$

$\cos\theta \neq 0$

$\theta = 90^\circ, 270^\circ$

$90 + k(180^\circ)\ k \in \mathbb{Z}$

b) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cos\theta \neq 0$

$\theta \neq 90, 270,$

$\theta \neq \pm 90 + 180(k),\ k \in \mathbb{Z}$

c) $\dfrac{\csc\theta}{\cos\theta}$

$= \dfrac{\frac{1}{\sin\theta}}{\cos\theta}$

$= \dfrac{1}{\sin\theta} \cdot \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\sin\theta(\cos\theta)}$

npv

$\sin\theta \neq 0$

$\theta = 0 + \pi(k)$

$k \in \mathbb{Z}$

OR

$\cos\theta \neq 0$

$\theta = \pm\frac{\pi}{2} + \pi(k)$

$k \in \mathbb{Z}$

d) $\dfrac{\sec\theta}{\sin\theta} = \dfrac{\frac{1}{\cos\theta}}{\sin\theta}$

$= \dfrac{1}{\cos\theta} \cdot \dfrac{1}{\sin\theta}$

$= \dfrac{1}{\cos\theta \sin\theta}$

npv:

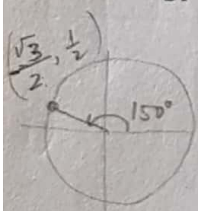
$\pi k,\ k \in \mathbb{Z}$

or

$\frac{\pi}{2} + \pi k,\ k \in \mathbb{Z}$

**5.** Verify each identity for the given value of $\theta$.

a) $\tan\theta \csc\theta \sec\theta = \sec^2\theta;\ \theta = 150^\circ$




$\tan 150 (\csc 150)(\sec 150)$

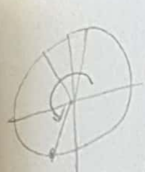
$\left(-\frac{1}{\sqrt{3}}\right)(2)\left(-\frac{2}{\sqrt{3}}\right)$

$= \frac{4}{3}$

$\sec^2(150)$

$= \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{4}{3}$



**b)** $\frac{\tan\theta\csc^2\theta}{\sec^2\theta} = \cot\theta; \boxed{\theta = \frac{4\pi}{3}}$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$\frac{\left(\tan\frac{4\pi}{3}\right)\left(\csc\frac{4\pi}{3}\right)^2}{\left(\sec\frac{4\pi}{3}\right)^2}$$

$$= \frac{(\sqrt{3})\left(\frac{2}{-\sqrt{3}}\right)^2}{\left(-\frac{2}{1}\right)^2}$$

$$= \frac{\sqrt{3}\left(\frac{4}{+3}\right)}{4}$$

$$= \frac{\cancel{4}\sqrt{3}}{3}\cdot\frac{1}{\cancel{4}}$$

$$= \frac{\sqrt{3}}{3}$$

$$= \frac{1}{\sqrt{3}}$$

$\cot\theta$

$\cot\left(\frac{4\pi}{3}\right)$

$$= \frac{\cos\frac{4\pi}{3}}{\sin\frac{4\pi}{3}}$$

$$= \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}}$$

**B**

**6.** Prove each identity in question 5.

$$LS = \frac{\tan\theta\csc^2\theta}{\sec^2\theta}$$

$$= \frac{\frac{\cancel{\sin\theta}}{\cos\theta}\cdot\frac{1}{\sin^{\cancel{2}}\theta}}{\frac{1}{\cos^2\theta}}$$

$$= \frac{1}{\cancel{\cos\theta}\sin\theta}\cdot\frac{\cos^{\cancel{2}}\theta}{}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$$= \cot\theta$$

$$= RS$$